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Abstract

The interbank market (IBM) is a crucial source of funding for banks, but market activity has shrunk considerably after Lehman Brothers' filled for bankruptcy in 2008. Consequently, the IBM market has also suffered from insufficient liquidity since this event. An important aim of this paper is to identify the conditions that have helped lead to an improvement in IBM liquidity via establishing a relationship. Although engaging in an interbank relationship is costly, it can stimulate lenders to participate in the IBM since relationships allow lenders to be more confident in their assessment about the borrower's quality. The model presented in this paper addresses the choice of a transaction or a relationship in lending using a single-choice setting prior to resolving the uncertainty about a borrower's quality. In such an uncertain setting, ambiguity-averse lenders are less confident than risk-averse ones: these ambiguity-averse lenders consider a range of distributions by which they behave cautiously when deciding on IBM participation. The model's equilibrium indicates under which conditions establishing a relationship is preferred to transactional banking, thus mitigating the market failure of liquidity shortages. Generally, relationship banking is preferred to transactional banking when the lender benefits in terms of confidence and precision when assessing a borrower's quality. It is especially important at times of deteriorating trust but becomes less evident the more severe the shock due to ambiguity is. For extreme levels, the improvement in a lender's confidence level is too low via relationships.

Keywords: Interbank Market; Financial Markets; Participation; Uncertainty; Ambiguity; Relationships.
JEL Codes: E43; G11; G01; G21.

1 Introduction

This paper presents a model that helps identify the conditions for improving participation and liquidity in the interbank market (IBM) by establishing a relationship between financial intermediaries. Interbank lending is possible by establishing stable lending relationships with other banks or by engaging in transactional banking. The IBM is crucial for the proper functioning of the modern financial system and the real economy.

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In times of crisis, such as the 2007-2008 financial crisis, increased uncertainty leads lending banks to question their assessment of a counterparty's risk profile. This loss in confidence in measuring risk profiles correctly causes banks to hoard liquidity rather than to provide liquidity in the IBM. As establishing a financial relationship has some known beneficial effects – such as to ensure mutual trust and confidence, and to overcome agency and informational problems[1] – it could potentially persuade lenders to provide a loan even though these lenders would not be willing to trade via a one-off transaction where no long term relationship exists.

In the case of the 2007-2008 financial crisis, large parts of the financial system required intermediation by central banks as the intermediation function of the IBM itself became inefficient. After the Lehman Brother's collapse, money markets came close to an entire freeze in developed countries, and as such, banks were then forced to borrow from central banks. Moreover, the effectiveness of a central bank's response determines the efficiency of the IBM functioning in crisis and non-crisis times, and in turn, the resilience of the IBM. Interestingly, the effectiveness of regulatory intervention depends on the uncertainty setting and on how preferentially connected a bank is, in addition to how many connections it has.

Relationships are inherent in the IBM; indeed, many unsecured interbank loans are agreed upon verbally and granted through repeated lending to relationship partners. Investigating the impact of establishing relationships in the IBM is a perfect setting as the empirical literature shows that it is particularly important at a time of deteriorating trust and increased uncertainty. During the 2007-2008 financial crisis, liquidity provision was scarce; but via relationships, a bank had better access to funds and at a lower price (e.g., Affinito, 2012; Temizsoy *et al*., 2015). Moreover, the type of banking determines the rate at which IBM loans are granted (e.g., Bech and Atalay, 2010). Although an IBM relationship is not random in the long-run, it materialises randomly and has advantages for both the lending and borrowing bank: trust, implicit insurance, and (soft) information acquisition on the one hand, and liquidity availability and more favourable terms of trade on the other hand (e.g., Hatzopoulos *et al*., 2016; Bräuning and Fecht, 2017). Together with liquidity insurance over time, this results in the continuation of bilateral lending, and hence, an interbank relationship between two banks. The literature highlights that banks have a higher reliance on IBM relationships due to a lack of confidence in credit profiles since the Lehman's collapse, and that financial turmoil is accompanied by a less familiar and more ambiguous environment.

On the more theoretical side, it is generally accepted that interbank relationships provide more favourable conditions than other types of bilateral banking, and this advantage persists and improves over time as it improves the lender's confidence in measuring a counterparty's credit risk[2]. Interbank relationships arise between two banks, and the lender in particular decides whether to lend with the intention of surviving in the long-run. Moreover, understanding banks' individual behaviour is crucial when considering the

[1] The advantage of information acquisition and informational advantages between a pair of banks that have a relationship is dealt with in Affinito (2012), Flannery (1996), and Rochet and Tirole (1996), among others. These papers show that relationship banking has an impact on the availability of liquidity and the pricing in the IBM.
[2] See Anand *et al*. (2015), and Freixas and Rochet (2008).

bigger picture: how does the structure of the financial system arise? Regulators should look at how preferentially connected a bank is besides understanding whether a bank is too connected to fail; they should determine the optimal level of relationship connections. Moreover, the effectiveness of a central bank's response depends on the uncertainty setting, and in turn, determines the efficiency of the IBM functioning during crisis and non-crisis times.

The key contribution of this paper is a model that investigates why a lending bank is persuaded to start lending although it would not be willing to trade via a one-off transaction where no long term relationship exists. Intuitively, when is relationship banking interesting in terms of IBM liquidity? To capture how confident IBM participants are, and how familiar the economic environment is, uncertainty[3] is modelled using measures of ambiguity plus risk when analysing the type of lending. Hence, this paper accounts for the inability to form a unique probability distribution of a counterparty's quality in financial systems. Specifically, the type of lending affects this inability, and the level of ambiguity perceived by lenders is, compared to transactional banking, reduced with relationship banking.

This paper suggests that the benefits of relationships prevail in an ambiguous environment, such as the 2007-2008 crisis and the current financial crisis in Russia, where the confidence in the counterparty's risk profile is low. I show that IBM liquidity is improved when more lenders participate and when these lenders are able to provide loans at a more favourable interest rate. Moreover, this paper reveals that liquidity mitigation with relationship banking is strong in crisis times as there is much to gain in confidence, either in ambiguity about the mean or in ambiguity about the variance. However, engaging in a relationship improves IBM liquidity only when the gain exceeds the cost. This means that in an ambiguous environment, it is not necessarily the case that relationship banking prevails: the effectiveness of engaging in a relationship depends on whether it allows lenders to alter their worst-case assessment of a borrower's risk profile enough to cover the costs related to this type of banking. Intuitively, for extreme levels of ambiguity, the improvement in lenders' confidence via relationships will be too low. As such, relationships do not improve IBM liquidity. Further, this paper's model also shows that relationship banking is preferred to transactional banking if the borrowing bank is complex and/or distressed.

One recommendation drawn from this paper's model is as follows: policy interventions should focus not only on improving confidence directly (i.e. via ambiguity reduction) but also indirectly by inducing relationship building in the IBM. This paper shows if and when establishing a relationship helps to mitigate the IBM failing and improve liquidity provision on it. The initial response by the Euro system to the recent financial crisis of 2007-2008 was to keep strategic reserves and to broaden eligible assets. In the rest of the world, policy interventions had a similar purpose as these tried to stimulate IBM participation, by broadening eligible collateral for instance.

While this paper's findings are relevant for regulators and central banks, they may also be of interest to other independent parties. The investment options for a lending bank include many products with varying degrees of quality, and thus, varying kinds of

[3] In this paper, I use *uncertainty* as a general term that captures both ambiguity and risk.

relationship building. Further, in various disciplines, situations arise where costly services are provided that affect one's ability to assess the counterparty's quality, so an agent's choice is influenced by ambiguity and/or risk.

A model that examines the choice of bank lending in a setting with uncertainty is presented in this paper and the analysis investigates the conditions under which establishing a relationship is preferred to transactional banking. Such a setting with ambiguity plus risk includes (as Ellsberg, 1961 has indicated) the behaviour of people when probabilities are unknown, in addition to the decisions made by them when probabilities are known. Hence, the degree of confidence is taken into account, and, as seen in financial markets, people typically face a complex environment where the probability distribution is unknown. Moreover, ambiguity varies over time, and this lack in confidence becomes crucial in times of crisis (see Drechsler, 2013; Flannery, Kwan and Nimalendran, 2013; and Rinaldi, 2009). As forming a precise opinion about a borrower's creditworthiness can ease liquidity distribution among banks, establishing relationships is particularly interesting when considering uncertainty. Hence, this paper is based on the rationale that attaching a probability to events in financial markets is easier with relationship banking because of the lender's improved confidence while assessing a counterparty's risk profile (e.g., Bräuning and Fecht, 2017). This improved confidence also means that banks do not lose mutual trust even in times of crisis. Although overcoming search frictions – due to informational asymmetries[4] – is also one of the advantages of relationships, this paper's model assumes that establishing a relationship improves one's confidence in assessing a counterparty's risk profile.

Using a framework with ambiguity and risk, the model considers a heterogeneous population of lenders who are either. After market clearing, the behaviour of lenders is explored with a particular interest on whether the type of banking can mitigate a liquidity shortage in the IBM or, more extremely, an IBM freeze where no lender is willing to provide loans. This study utilises an ambiguous environment by considering a range of distributions, which includes the people who do not have a unique and objective probability distribution. On the other hand, when confronted with risk people consider one distribution of probabilities. The heterogeneous set of lenders represents the range of IBM segments, from fully confident to not confident in the assessing of a borrower's quality. Risk-averse lenders do not face a loss in confidence, but ambiguity-averse lenders do and they are unable to form a unique probability distribution of a counterparty's credit risk. For example, when comparing IBM participants in the cross-border and domestic markets, lenders can be seen to be more confident about their assessment of a counterparty's quality when it involves a domestic one[5]. Moreover, risk-averse lenders are considered as experienced agents, and, therefore, do not face ambiguous returns[6].

[4] The theoretical work by Duffie *et al.* (2017) investigates the impact of an improvement in transparency on the efficiency of over-the-counter markets with search frictions.

[5] Mistrulli (2011) differentiates, for instance, between the cross-border and domestic IBM based on information problems, and states that the cross-border IBM shrunk due to the loss of confidence after Lehman Brother's collapse.

[6] Easley and O'Hara (2010) propose linking microstructure to ambiguity because an ambiguity-averse agent faces an ambiguous return from participating in the market. The authors propose a set of lenders including sophisticated and unsophisticated agents in a single-shot model to resolve uncertainty. Moreover, they state that risk-averse agents might be inherently ambiguity averse but encounter, due to their experience with market workings, nearly no ambiguity.

The analysis' starting point is the multiple-priors model of Gilboa and Schmeidler (1989) for analysing the optimal investment portfolio in liquidity and IBM loans. Whereas a borrowing bank is in need of liquidity after a distributional shock, a lending bank has excess liquidity; this liquidity mismatch is resolved in the IBM. The participation decision by lenders is made in a single-shot-choice setting prior to the resolution of uncertainty about the borrower. In light of this paper's central research question, the difference in the type of aversion is stipulated in a preference model. The two types of lenders have CARA-normal preferences: *i*) risk-averse lenders that are modelled as having a unique probability distribution and the ability to assess counterparty quality and *ii*) ambiguity-averse lenders that do not have a unique prior and cannot make a precise assessment of the borrower's default. A borrower's (partial) default and the associated costs are represented as a cost to the return, which are called participation costs, as they represent the costs associated with the liquidity supply to the IBM. As with the type of banking, the expected rate of return on issued loans differs with the type of lender. Further, establishing a relationship goes hand in hand with costs: these are investments made to reduce vulnerability to economic events and are costly for a lending bank. These costs are called relationship costs[7]. The types of banking differ in terms of perceived ambiguity; with relationship banking, the perceived ambiguity is reduced compared to transactional banking. Hence, the aversion to subjective uncertainty is lower with relationship banking and the ambiguity-averse lender is more equipped to form a unique probability distribution. Intuitively, lenders become more confident about their assessment of the borrower's credit quality and the probability of default. When engaging in a relationship, banks behave more confidently and do not lose mutual trust even in times of crisis. On the other hand, transactional banking represents a single transaction without improved confidence and (soft) information acquisition.

Optimising each lender's wealth according to the type of banking shows which lenders engage in IBM lending for a certain interbank rate. Moreover, the volume of surpluses that is provided to the IBM differs depending on the type of lender and the type of banking. For each banking type there is either full or partial participation, meaning that either the entire pool of lending banks participates or the pool of ambiguity-averse lenders do not engage in IBM lending. If relationship banking improves the perceived ambiguity sufficiently in terms of mean and/or variance, relationship banking improves IBM liquidity. Otherwise, transactional banking is the most appealing way of extending a loan. For risk-averse lenders, in contrast to ambiguity-averse ones, the banking type does not have an influence on the liquidity provision as these lenders only take risk into account.

Participation by the ambiguity-averse lender depends on the type of lending: there is no overall effect, and participation depends on the parameter values of the relationship, i.e. the extent of the costs and benefits of building a relationship. For the risk-averse lender however, relationship banking does not improve participation because a risk-averse lender only accounts for risk. This paper's model is based on the economic rationale that

[7] One can imagine the loss when a negatively correlated bank pair is affected by a structural break. Such a break causes an aggregate liquidity deficit, and hence, impairs the liquidity insurance mechanism. Further, other explanations for relationship costs are the investments made in screening, negotiating, and monitoring to acquire information and maintain information up to date about the counterparty.

relationships allow lenders to be more confident in their assessment of a counterparty's risk profile, which relates directly to ambiguity. Whereas ambiguity-averse lenders have maxmin preferences and behave cautiously, risk-averse ones maximise utility while considering a unique probability distribution of participation costs. Ambiguity-averse lenders maximise the worst-case utility, which is determined by minimising the expected utility over a set of distributions that such ambiguity-averse lenders consider are possible[8]. Even more precise, the set of distributions that an ambiguity-averse lender considers is smaller with relationship banking than with transactional banking. Similarly, the worst-case utility is more severe and lower with transactional banking.

The equilibrium outcome depends crucially on a borrower's preferences and is found by minimising the interbank rate. While the lender tries to meet demand and proposes a different interbank rate according to the type of banking, the borrowing bank decides which type of banking will prevail, and this choice is driven by funding costs. The prevailing equilibrium outcome depends on the proportion of ambiguity-averse lenders, the perceived ambiguity, the relationship costs, the amount of risk, and the borrowers' demand.

The banking system is quite different from other industries, and due to its specific structure, requires separate insights on the role of relationships. On this topic however, the theoretical literature is quite scarce. Unlike other industries, competitors do not gain from the failure of another bank. Rather, bank failures generate negative externalities for other banks: less confidence in the financial system's stability, and losses due to the interbank exposures to failed banks, among others. Further, the IBM is an over-the-counter market that works quite differently from the bank-firm financial market systems. In the latter, the ambiguity problem arises because the lender may have insufficient confidence in his knowledge about the counterparty and it may not be efficient to gain such expertise. By contrast in the IBM, the ambiguity problem arises because banks are already unusually opaque compared to non-bank firms and this opaqueness grows in crisis times (Flannery, Kwan and Nimalendran, 2013). Another characteristic of the IBM is that a number of similar financial contracts exists, meaning that a bank, for instance, borrows from multiple lenders at the same time. Furthermore, the empirical literature suggests that interbank lending relationships or bank-bank relationships differ from traditional bank-firm relationships. One reason is the important liquidity-sharing motives related to overcoming information asymmetries (e.g. Cocco, Gomes and Martins, 2009).

The paper is organised as follows. Section 2 summarises how this paper's model fits into the literature, whereas section 3 describes a model in financial markets inspired by portfolio choice theory and starting from the multiple-priors model of Gilboa and Schmeidler (1989). In Section 4, the optimisation problem is solved while imposing market clearing and borrower's preferences. A unique market equilibrium is the result. Section 5 shows the determinants of the equilibrium while focusing on the difference between transactional and relationship banking. Moreover, together with these determinants, empirical predictions are made.

[8] The papers of Easley and O'Hara (2009), and Easley and O'Hara (2010) are examples of participation models where an investor's demand for risky assets is optimised.

2 Literature Review

This paper adds to the scarce theoretical literature on relationships in the IBM with an uncertainty model that accounts for ambiguity plus risk[9]. The literature lacks a formal lending theory where the type of banking can explain the differences in interbank rates that exist in normal times and during a crisis, as seen empirically. IBM participants understand each other's credit risk profile better with relationships and are more confident about their assessment with a relationship (e.g. Affinito, 2012; Cocco, Gomes and Martins, 2009; and Schoar, 2012). Nevertheless, Blasques, Bräuning and Lelyveld (2015) investigate relationships in a setting of uncertainty, with a focus on the impact of risk, and then estimate model parameters using data from the Dutch interbank market.

Typically, the literature deals with relationships in the long run and focus on the impact of monetary policy without accounting for the choice in type of lending or how relationships originate. One example is the paper by Chiu and Monnet (2015). Moreover, investigating the network of IBM relationships is interesting for stress testing. In the paper of Anand *et al*. (2015) a network structure is built via the minimum-density method based on the notion that relationships are costly to add and to maintain. With this economic rationale, the authors provide an alternative to the maximum-entropy approach. Further, with regard to network formation, Finger and Lux (2017) propose a stochastic actor-oriented model for network formation that arises as past relationships determine liquidity provision by lenders. Also, simulation modelling is used for network formation such that the dynamic evolution and the impact on IBM stability can be investigated. (e.g. Liu *et al*., 2017; Xu *et al*., 2016).

This paper's contribution to the literature mainly comes from its focus on the establishment of a financial relationship and on its origin. In particular, one might be curious about the effect of balance sheet contractions, as seen during the 2007-2008 crisis, on IBM participation, volume, and prices with relationships and plain transactions. Regulators should look at how preferentially connected a bank is along with establishing whether a bank is too connected to fail. When banks meet by random matching, they can either trade via a one-off transaction or engage in a relationship. For instance, Babus (2016) looks at the degree of connectivity to differentiate between equilibria with and without contagion. This author's model fits into the wide literature on determining the impact of interbank linkages on the fragility of the financial system.

This paper is further related to a number of others. The existence of stable interbank relationships, which remained throughout the financial crisis, is recognised in the empirical literature (Fricke and Lux, 2015; Hatzopoulos *et al*., 2016; Temizsoy *et al*., 2015)[10].

[9] People typically face a complex environment in financial markets (e.g. Flannery, Kwan and Nimalendran, 2013; Rinaldi, 2009), where they are not told the probability distribution and the environment is already unusually opaque, compared to a non-bank environment. In particular, markets in financial turmoil tend to be accompanied by a less familiar and more ambiguous environment.

[10] The notion of random matching for the initiation of relationships is dealt with in, for instance, Iori *et al*. (2015), and Hatzopoulos *et al*. (2016). While Temizsoy *et al*. (2015) show that relationships arise fairly randomly, Bräuning and Fecht (2017) show that the persistence of relationships is not random and depends on previous lending decisions.

Moreover, the literature also indicates that pricing differs for transactional and relationship banking (e.g. Bräuning and Fecht, 2017).

The informational advantages of a relationship are dealt with in a number of papers such as Affinito (2012), Flannery (1996), and Rochet and Tirole (1996), among others[11]. Together with liquidity insurance over time, maintaining a relationship results in the continuation of bilateral lending and hence, the interbank relationship between two banks continues. One example is the paper of Cocco, Gomes and Martins (2009), who suggest that relationships arise between less correlated bank pairs. Moreover, the authors discuss liquidity insurance in the light of market frictions such as transaction and information costs. Further, the paper of Castiglionesi and Wagner (2013) studies the incentives for liquidity insurance, and it shows that such an outcome is efficient.

Bank loans in general, and failure in the IBM during crisis times, have already been extensively dealt with in the literature. As became prominent during the 2007-2008 crisis, a well-functioning IBM is critical for the efficiency and effectiveness of the whole financial system, monetary policy implementations, and credit supply to the real economy (e.g. Acharya, Gromb and Yorulmazer, 2012; Freixas, Martin and Skeie, 2011; Kharroubi and Vidon, 2009, and Montagna and Lux, 2017). During the 2007-2008 financial crisis, banks were confronted with unfulfilled liquidity needs and the IBM's intermediation function became inefficient, making regulatory interventions necessary. The effectiveness of such interventions determines the efficiency of the IBM functioning in crisis and non-crisis times, and in turn, the resilience of the IBM[12].

3 The Model

In this section, the model is described where the type of lender and the type of banking both determine how much liquidity is available in the interbank market (IBM). The existence of an IBM is shown in the literature as follows (e.g. Bhattacharya and Gale, 1987; Freixas and Holthausen, 2004). Due to a distributional shock, such as the unexpected withdraw of deposits, banks face liquidity shortages and access the IBM to fulfil liquidity needs. Lending banks can fulfil these needs through transactional or relationship banking. The IBM is the place whose banks with heterogeneous liquidity preferences match with each other. Participants who are in need of liquidity look for a counterparty with a liquidity surplus[13]. In this paper there are $I + 1$ possible investment

[11] Although overcoming search frictions is also one of the advantages of relationships, this paper's model assumes that attaching a probability to events in financial markets is easier with relationship banking because of the lender's improved confidence in assessing a counterparty's risk profile.

[12] Allen and Gale (2000) analyse the structure, efficiency, and resilience of the IBM. In particular, their model, with idiosyncratic liquidity needs and private information, shows how contagion arises after experiencing an exogenous aggregate liquidity shock. See also Renard and Wuyts (2014) for an analysis on the effectiveness of policy interventions in the IBM. Specifically, resolving an IBM freeze is discussed in an environment of uncertainty that considers ambiguity in addition to risk.

[13] Seminal papers, such as Bhattacharya and Gale (1987), and Freixas and Holthausen (2004) provide a rationale for the existence of an IBM. One interpretation for the heterogeneous liquidity preferences is that the withdrawal of deposits may be above or below expectations. Another example is the heterogeneous investment opportunities:

opportunities: one risk-free asset and I risky IBM loans denoted by $i = 1, ..., I$. Each loan's future value is represented by a random variable where its values are independently and normally distributed.

Using a heterogeneous lender model, how market-clearing in an IBM arises is first dealt with and the associated pricing that depends on banking type is discussed. Subsequently, the determinants of the equilibrium are investigated. Then, empirical predictions are examined relevant with respect to how liquidity shortages can arise in the IBM, and in the extreme, evolve into a IBM freeze. While it may be relatively unimportant in normal times, during a financial crisis the complexity of the environment, a loss in confidence in the financial system, and more specifically, the effect of ambiguity may be of an extent that cannot be explained by risk alone. I then explore whether the type of banking, transactional (T) and relationship (S) banking, can mitigate this market failure.

The IBM framework is inspired by the multiple-priors model of Gilboa and Schmeidler (1989), the portfolio choice theory in financial markets, and the participation models of Easley and O'Hara (2009; 2010). The importance of an ambiguous setting is in its reinterpretation of subjective expected utility (SEU) theory: ambiguity aversion can, for example, substitute for high risk-aversion levels. Moreover, a change in ambiguity results in a relatively large effect compared to a change in riskiness. Prior to the resolution of uncertainty, in terms of either ambiguity or risk, the choice of lending volume is made in a single-shot-choice setting. A lending bank is randomly matched with a borrower in need of liquidity. The literature explains relationship formation as the result of mutual trust and confidence, informational advantages or liquidity insurance between a pair of banks[14]. With relationship banking, as opposed to transactional banking, priors are formed more precisely. These more precise priors imply that the lending bank is more confident and better equipped to assess the credit exposures of its counterparty with a relationship than with transactional banking.

3.1 Lender's decision problem

The IBM is a perfectly competitive market with random matching between lending and borrowing banks[15]. Distributional shocks lead to a pool of lending and borrowing banks. While the former are confronted with excess liquidity, the latter are in need of liquidity. Without any aggregate liquidity shock, the aggregate liquidity surplus is suf-

while one bank lacks liquidity for an investment opportunity, another one has sufficient liquidity available but has no investment opportunity at its disposal.

[14] The motivation for discussing relationship banking next to transactional banking in a random matching framework is found in previous studies, such as those of Lux (2015) and Hatzopoulos *et al*. (2016) who show that choosing an initial trading partner is fairly random.

[15] An IBM can be viewed as a random network on a daily scale, and the structure becomes non-random for longer periods (see Iori *et al*., 2015). Further, Hatzopoulos *et al*. (2016) and Temizsoy *et al*. (2015) state that relationships arise fairly randomly, but Bräuning and Fecht (2017) show that the persistence in relationships is not random, and depends on previous bilateral lending.

ficient to fulfil borrowers' liquidity needs. The J lenders have CARA preferences and a risk-aversion parameter equal to $\alpha = 1$ by which its utility of wealth w_j is

(1) $$U(w_j) = -\exp(-w_j)$$

For ease of notation, bank-specific subscripts are not used in the remainder of the text; therefore, the utility of wealth w is denoted as $U(w) = -\exp(-w)$.

Two types of lenders with CARA-normal preferences provide liquidity to the IBM: ambiguity-averse (AA) and risk-averse (RA) lenders. The former choose the lending volume more conservatively because they are not confident in correctly measuring the counterparty's quality. They cannot precisely grasp the costs in the same way as RA lenders, and therefore, require an additional ambiguity premium for participating in an ambiguous environment. This paper considers ambiguity aversion as the inability to form a single probability, whereas risk-averse people are able to precisely assess a counterparty's creditworthiness. More specifically, the AA lenders consider multiple priors, a set of possible distributions as in Gilboa and Schmeidler (1989), and choose to lend cautiously. In the light of precautionary behaviour, AA lenders tend to maximise utility over the set of priors by maximising the worst-case scenario. Alternatives of static preference models typically used in financial markets research are the smooth ambiguity model (Klibanoff *et al.*, 2005) and the multiplier utility model (Hansen and Sargent, 2001).

The lender's budget constraint is:

(2) $$w = m + \sum_i x^i$$

where m represents the amount of money and x^i the quantity of a risky IBM loan i with $i = 1, ..., I$. It is assumed that the quantity x^i is positive as it represents the amount invested in loan i. Such a loan offers a return of $R^i - \tilde{\varepsilon}^i$ after one period, where R^i is the market interest rate and $\tilde{\varepsilon}^i$ is a random participation cost, which can be solvency costs when the borrower fails to repay the loan (partly) or a negative impact on balance sheets due to the failure of borrowers (e.g., Iori *et al.*, 2006) for example. The idea is that IBM participation is costly for lending banks, and therefore, $\tilde{\varepsilon}^i$ represents the participation costs incurred by issuing loan i.

The aggregate lending volume is determined by the population's composition of risk- and ambiguity-averse lending banks. This composition thus determines participation in the IBM. Section 3.4 then shows how lending volume determines the interbank rate. While the effect of a lender's type might be relatively small in normal times, during a financial crisis there may be a significant difference in preferences.

Consider first a proportion of $1-\mu$ risk-averse banks that are confident in measuring the counterparty's creditworthiness correctly: they are confident about the odds and form a unique prior about the participation costs. A RA lender takes the *i.i.d.* distribution $\tilde{\varepsilon}^i(\hat{\varepsilon}^i, \hat{\sigma}^i)$ into account and considers the return on an interbank loan $R^i - \hat{\varepsilon}^i$. By contrast, a proportion of μ ambiguity-averse banks are not confident in their ability to assess a borrower's quality: they are not confident about the true distribution, and consider a set of possible means and variances for the participation costs. These AA banks take a set of distributions into account when deciding, one distribution for each combination of a possible mean and a possible variance. Hence, AA lenders consider a range of expected means $(\overline{\varepsilon}_1^i, \overline{\varepsilon}_2^i, \ldots, \overline{\varepsilon}_N^i)$ and variances $(\overline{\sigma}_1^i, \overline{\sigma}_2^i, \ldots, \overline{\sigma}_N^i)$ with a normal distribution of payoffs. These lenders consider the set $\theta^i = (\theta_1^i, \theta_2^i, \ldots, \theta_N^i)$ with $n = N^2$ elements. This set represents the combinations of means and variances that form a convex set of priors, where the number of possible combinations reflects a bank's level of ambiguity. The two sets of possible means and variances contain values above and below $\hat{\varepsilon}^i$ and $\hat{\sigma}^i$, such that the true values are convex combinations of the extreme values considered by the AA lenders.

3.2 Transactional and relationship banking

Interbank lending is possible with transactional (T) and relationship banking (S). The two types of banking differ in the confidence a lender has in assessing the counterparty's credit risk profile. Mitigating informational problems is at the heart of a bank's existence. Relationship banking aims to resolve confidence issues involved in measuring credit risk correctly, meaning a more precise assessment of the borrower's quality. Moreover, establishing relationship banking helps to ensure mutual trust, to overcome agency and informational problems since soft and private information about counterparties is gained. Further, in times of financial turmoil, liquidity provision can remain sufficient in the IBM through relationship banking as it enables a lender to maintain trust. Interestingly, the literature highlights that liquidity – through relationship banking – flows from healthier banks to those banks most severely hit by crisis. For example, after the Lehman's collapse, there was a greater reliance on relationship banking due to a lack of confidence in credit profiles (e.g. Affinito, 2012; Bräuning and Fecht, 2017). Although a relationship based on confidence is beneficial, with transactional banking the focus is placed on the transaction without gaining preciseness in assessing credit risk and the vulnerability to economic events.

In this paper, the above motivation is followed: establishing a relationship implies a better understanding of a counterparty's risk profile. So relationship banking allows lenders to reduce ambiguity compared to transactional banking: the set θ^{Si} considered by an AA lender is smaller than the set θ^{Ti}. The worst-case scenario imagined is limited by the lending bank's increased confidence and activity of collecting bank-specific information. With relationship banking, trust and information acquisition ensure the following maximum values: $\overline{\varepsilon}_{\max}^{Si} < \overline{\varepsilon}_{\max}^{Ti}$ for participation costs and $\overline{\sigma}_{\max}^{Si} < \overline{\sigma}_{\max}^{Ti}$ for the maximum variance. Note that when a lender considers the smaller set of relevant mean-variance parameters θ^{Si}, and is less ambiguous, it does not insure that the borrower is a good counterparty; it only rules out some of the extreme situations imagined by the lending bank.

3.3 The banking type choice

A borrowing bank prefers to pay as little as possible for acquiring funds, and accordingly, chooses the type of banking to employ. Whether it engages in transactional (T) or relationship (S) banking, the prevailing banking type in equilibrium is determined by the lowest rate for an interbank loan, i.e. the worst-case rate. The interbank rate is set by lenders and reflects the rate required to supply excess liquidity to the IBM. The willingness to provide liquidity to the IBM increases with the interbank rate, but with an increasing rate the borrower's funding costs increase.

The profitability of a loan is determined by relationship costs and how averse a lender is to the induced subjective uncertainty. Whereas a lending bank does not pay a cost with the transactional banking, it does pay a cost on the loan amount with relationship banking. The costs that accompany relationship banking are denoted $c^i = c(x^i)$. These costs represent the investments made by the lending bank for establishing a relationship, such as the screening and negotiating, and consequently, they act to reduce the vulnerability to economic events[16]. These investments are increasing in size and complexity, but they are capped as savings in investments are gained. The relationship costs are therefore lower when a borrower is large and/or complex. A loan's profitability is represented as $R^i - \overline{\varepsilon}^{Si}_{\max} - c^i$ for relationship banking. With transactional banking however, a loan's profitability is $R^i - \overline{\varepsilon}^{Ti}_{\max}$. Regarding participation costs, the worst case imagined with relationship banking $\overline{\varepsilon}^{Si}_{\max}$ is lower than with transactional banking $\overline{\varepsilon}^{Ti}_{\max}$ because a lender's aversion is tempered.

3.4 Interbank market participation

Lenders have wealth w and choose to invest in the risk-free asset m and risky IBM loans. After the lender's investment decision $(m, x^1, ..., x^I)$, the future random wealth $\tilde{w}$ is:

$$\tilde{w} = m + \sum_i (R^i - \hat{\varepsilon}^i - I^i c^i)x^i$$

where $I^i = 1$ for relationship banking and $I^i = 0$ for transactional banking.

Two types of lenders with CARA-normal preferences provide liquidity to the IBM: ambiguity-averse (AA) and risk-averse (RA) lenders. The latter are Subjective Expected Utility (SEU) decision makers as in Savage (1954). These RA lenders maximise expected utility over wealth w while being confident about their assessment of counterparty quality: they are confident about knowing the true probability distribution of the participation costs. In particular, for a given IBM rate, RA lenders maximise the expected utility of the random future wealth $\tilde{w}$ according to the true priors $(\hat{\varepsilon}, \hat{\sigma})$. For CARA-normal preferences, this optimisation problem is equivalent to the following:

[16] For relationship banking, the following maximum values are considered: $\overline{\varepsilon}^{Si}_{\max} < \overline{\varepsilon}^{Ti}_{\max}$ for the participation costs and $\overline{\sigma}^{Si}_{\max} < \overline{\sigma}^{Ti}_{\max}$ for the maximum variance.

$$\max_{m_{RA},x^i_{RA}} f(m_{RA},x^1_{RA},\ldots,x^I_{RA}) \tag{3}$$

$$\max_{m_{RA},x^i_{RA}} \sum_i \left[m_{RA} + (R^i - I^i c^i - \hat{\varepsilon}^i) x^i_{RA} - \frac{1}{2}\hat{\sigma}^i (x^i_{RA})^2 \right] \tag{4}$$

subject to

$$\begin{gathered} \bar{m}_{RA} \geq m_{RA} + x^1_{RA} + \ldots + x^I_{RA} \\ \bar{m}_{RA},\bar{m}_{RA} \geq 0 \\ \forall i{:}\, x^i_{RA} \geq 0 \\ \forall i{:}\, R^i - \hat{\varepsilon}^i - I^i c^i \geq 0 \end{gathered} \tag{5}$$

where, without loss of generality, the risk-aversion parameter has been equalised to one and the subscript RA is added to denote the parameters and the variables that are specific for the RA lenders. The amount of excess liquidity available for IBM lending, i.e. the lending bank's surplus, is denoted by $\bar{m}_{RA}$, and it is the upper bound for investing in money and IBM loans. The first constraint of the optimisation problem above states that the sum of the investments in the risk-free asset and the IBM loans cannot be larger than the lending bank's excess liquidity available. The second constraint prohibits a negative amount of liquidity assets and assumes that RA lenders have a liquidity surplus. Further, shorting is prohibited in the third constraint, and a positive return on the loan is assumed in the fourth constraint. Optimising, by using the Kuhn-Tucker conditions, results in a supply function for IBM loan i equal to:

$$x^{*i}_{RA}(R^i) = \begin{cases} 0 & \mathit{if}\ R^i \leq 1 + \hat{\varepsilon}^i + I^i c^i; \dfrac{R^i - \hat{\varepsilon}^i - 1 - I^i c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \dfrac{R^i - \hat{\varepsilon}^i - 1 - I^i c^i}{\hat{\sigma}^i} & \mathit{if}\ R^i > 1 + \hat{\varepsilon}^i + I^i c^i; \dfrac{R^i - \hat{\varepsilon}^i - 1 - I^i c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \bar{m}_{RA} & \mathit{otherwise}, \end{cases} \tag{6}$$

which shows that there are three possibilities: no participation in the IBM, supplying some of the excess liquidity to the IBM, and IBM lending being bounded by the lending bank's available funds. While relationship banking helps to limit the ambiguity perceived by a lender, it has no impact on the level of risk. Hence, for a RA bank the parameters ε^i and σ^i apply no matter which type of banking is employed. In more detail, the RA lender's supply function is then:

$$x^{*i}_{RA}(R^i) = \begin{cases} 0 & \mathit{if}\ \text{for } I^i = 0{:}\, R^i \leq 1 + \hat{\varepsilon}^i, \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} < \bar{m}_{RA} \\ & \quad \text{for } I^i = 1{:}\, R^i \leq 1 + \hat{\varepsilon}^i + c^i, \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} & \mathit{if}\ I^i = 1, R^i > 1 + \hat{\varepsilon}^i + c^i, \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} & \mathit{if}\ I^i = 0, R^i > 1 + \hat{\varepsilon}^i, \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \bar{m}_{RA} & \mathit{otherwise}, \end{cases}$$

which makes a clear distinction between supply with transactional and supply with relationship banking.

The AA lenders are ambiguity-averse decision makers in the sense of Gilboa and Schmeidler (1989), meaning that they do not have unique priors like an RA lender does. An AA lender's beliefs are characterised by a set of distributions, or more specifically, by the set of possible means and variances $\theta^i = (\theta_1^i, \theta_2^i, \ldots \theta_N^i)$. Optimisation of the expected utility of the random future wealth $\tilde{w}$ is found by considering max-min preferences: after calculating the worst-case expected utility over the set θ^i – which represents how confident a lender is about the measurement of the counterparty's quality – the worst-case utility is maximised while choosing the investment portfolio of money and risky IBM loans. In particular, an AA lender evaluates the expected utility of wealth for each combination of θ^i and considers the worst case due to his precautionary behaviour. Subsequently, the AA lender's decision problem is then solved by considering the worst case while choosing the investment portfolio. Intuitively, this type of lender makes decisions in a conservative way because the participation costs are not grasped with the same preciseness as RA lenders; therefore, an additional premium is required to be motivated sufficiently. For a certain level of R, the optimisation problem for an AA lender is as follows:

$$\begin{aligned} &\max_{m_{AA}, x^i_{AA}} \min_{\theta^i} f(m_{AA}, x^1_{AA}, \ldots, x^I_{AA}) \\ &\max_{m_{AA}, x^i_{AA}} \min_{\theta^i} \sum_i \left[m_{AA} + (R^i - I^i c^i - \hat{\varepsilon}^i) x^i_{AA} - \frac{1}{2} \hat{\sigma}^i (x^i_{AA})^2 \right] \end{aligned} \tag{7}$$

subject to

$$\begin{aligned} &\bar{m}_{AA} \geq m_{AA} + x^1_{AA} + \ldots + x^I_{AA} \\ &m_{AA} \geq 0 \\ &\forall i: x^i_{AA} \geq 0 \\ &\forall i: R^i - I^i c^i - \hat{\varepsilon}^i \geq 0 \end{aligned} \tag{8}$$

where, without loss of generality, the ambiguity aversion parameter has been equalised to one and the subscript AA is added to denote the parameters and variables that apply for the AA lenders. The AA lenders have an amount $\bar{m}_{AA}$ available for IBM lending, which is not necessarily supplied to the IBM. This amount represents the upper bound for investing in money and IBM loans. The constraints above are similar to those for the RA lenders. The first constraint states that the aggregate of IBM lending and investing in the risk-free asset should not exceed a lending bank's available funds. The second constraint prohibits a negative amount of liquidity and assumes that an AA lender has excess liquidity. Furthermore, shorting is prohibited in the third constraint and a positive return on the IBM loan is assumed in the fourth constraint. The optimisation problem for AA lenders is solved in two steps as an AA lender behaves cautiously and considers the worst case. Then portfolio is chosen such that the exposure corresponding to the worst case is limited. First, the lender considers the worst-case scenario by minimising the expected utility over the set $\theta^i = (\theta_1^i, \theta_2^i, \ldots \theta_N^i)$, which implies that the maximum of the participation costs and the maximum variance are the crucial parameters. Indeed, the worst return on loan i is driven by $\bar{\varepsilon}^{i}_{\max}$ and $\bar{\sigma}^{i}_{\max}$ because these parameters minimise the AA lender's utility for the given set of priors. In a second step, the lender selects the investment portfolio such that the expected utility of wealth associated to the worst case is maximised. The optimisation problem is solved by using Kuhn-Tucker conditions and:

$$(9)\, x^{*i}_{AA}(R^i) = \begin{cases} 0 & if\ R^i \le 1 + \bar{\varepsilon}^{i}_{\max} + I^i c^i; \dfrac{R^i - \bar{\varepsilon}^{i}_{\max} - 1 - I^i c^i}{\bar{\sigma}^{i}_{\max}} < \bar{m}_{AA} \\ \dfrac{R^i - \bar{\varepsilon}^{i}_{\max} - 1 - I^i c^i}{\bar{\sigma}^{i}_{\max}} & if\ R^i > 1 + \bar{\varepsilon}^{i}_{\max} + I^i c^i; \dfrac{R^i - \bar{\varepsilon}^{i}_{\max} - 1 - I^i c^i}{\bar{\sigma}^{i}_{\max}} < \bar{m}_{AA} \\ \bar{m}_{AA} & otherwise \end{cases}$$

is an AA lender's supply function for the IBM loan i. While banking type has no influence on the level of risk, the relevant ambiguity parameters differ according to the type of banking, and consequently matters for AA lenders. For each IBM loan i, the worst-case scenario applies when AA lenders consider the maxima, but the relevant parameters $\bar{\varepsilon}^{i}_{\max}$ and $\bar{\sigma}^{i}_{\max}$ differ for transactional and relationship banking. The participation costs are $\bar{\varepsilon}^{Si}_{\max}$ and $\bar{\varepsilon}^{Ti}_{\max}$ for, transactional and relationship banking respectively, and the variances are denoted $\bar{\sigma}^{Si}_{\max}$ and $\bar{\sigma}^{Ti}_{\max}$, respectively. These features allow the AA lenders to specify their supply function as follows:

$$x^{*i}_{AA}(R^i) = \begin{cases} 0 & if\ R^i \le 1 + \bar{\varepsilon}^{Ti}_{\max} \text{for } I^i = 0; \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} < \bar{m}_{AA} \\ & R^i \le 1 + \bar{\varepsilon}^{Si}_{\max} + c^i \text{for } I^i = 1; \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} < \bar{m}_{AA} \\ \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} & if\ R^i > 1 + \bar{\varepsilon}^{Ti}_{\max} \text{for } I^i = 0; \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} < \bar{m}_{AA} \\ \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} & if\ R^i > 1 + \bar{\varepsilon}^{Si}_{\max} + c^i \text{for } I^i = 1; \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} < \bar{m}_{AA} \\ \bar{m}_{AA} & otherwise \end{cases}$$

where $I^i = 1$ for relationship banking, and $I^i = 0$ for transactional banking. The supply is, however, not necessarily represented as increasing.

For a given IBM rate R^i, the supply functions of AA and RA lenders, as derived above, are presented in Lemma 1[17]. For the remainder of the paper, one should note a number of important features represented in this lemma. First, it shows that both of the supply functions consist of three parts: *i*) non-participation or no lending, *ii*) lending some

[17] Note that for AA banks the supply $\dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}}$ is not necessarily smaller than $\dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}}$. As will be discussed, this depends on the parameter values.

excess liquidity, and *iii*) lending all of the excess liquidity[18]. The case *iii*) shows that, although the lender is willing to provide more liquidity, funds are limited. Cases *i*) and *ii*) are the most interesting for this paper; therefore, the non-participation case and the case of intermediate participation are focused on[19]. In particular, the parameters values are such that the supply of IBM loans is (strictly) smaller than $\bar{m}_{AA}$ and $\bar{m}_{RA}$, for AA and RA lenders, respectively.

Lemma 1 *Given R^i, the aggregate supply function of the IBM loan i can be found by considering IBM lending provided by:*

- *the RA banks, denoted $x^{*i}_{RA}(R^i)$:*

$$x^{*i}_{RA}(R^i) = \begin{cases} 0 & \text{if for } I^i = 0: R^i \leq 1 + \hat{\varepsilon}^i, \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} & \text{for } I^i = 1: R^i \leq 1 + \hat{\varepsilon}^i + c^i, \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} & \text{if } I^i = 1, R^i > 1 + \hat{\varepsilon}^i + c^i, \dfrac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i} < \bar{m}_{RA} \\ \bar{m}_{RA} & \text{if } I^i = 0, R^i > 1 + \hat{\varepsilon}^i, \dfrac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} < \bar{m}_{RA} \\ & \text{otherwise} \end{cases} \tag{10}$$

- *the* AA *banks, denoted $x^{*i}_{AA}(R^i)$:*

$$x^{*i}_{AA}(R^i) = \begin{cases} 0 & \text{if } R^i \leq 1 + \bar{\varepsilon}^{Ti}_{\max} \text{ for } I^i = 0; \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} < \bar{m}_{AA} \\ \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} & R^i \leq 1 + \bar{\varepsilon}^{Si}_{\max} + c^i \text{ for } I^i = 1; \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} < \bar{m}_{AA} \\ \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} & \text{if } R^i > 1 + \bar{\varepsilon}^{Ti}_{\max} \text{ for } I^i = 0; \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} < \bar{m}_{AA} \\ \bar{m}_{AA} & \text{if } R^i > 1 + \bar{\varepsilon}^{Si}_{\max} + c^i \text{ for } I^i = 1; \dfrac{R^i - \bar{\varepsilon}^{Si}_{\max} - 1 - c^i}{\bar{\sigma}^{Si}_{\max}} < \bar{m}_{AA} \\ & \text{otherwise} \end{cases} \tag{11}$$

Second, AA lenders make decisions cautiously because they doubt their assessment of participation costs. Hence, AA lenders' decision to participate is made by taking the highest possible participation costs into account, meaning that these lenders consider the maximum mean $\bar{\varepsilon}^i_{\max}$ only instead of the full set of possible means and variances.

[18] The function for investing in the risk-free asset is as follows. For a given $R^i, m^*_{RA}(R^i) = \bar{m}_{RA} - \sum_i x^{*i}_{RA}(R^i)$ applies for RA lenders, and $m^*_{AA}(R^i) = \bar{m}_{AA} - \sum_i x^{*i}_{AA}(R^i)$ for AA lenders.

[19] Including case *iii*) is technically easy to implement but is of little value as it does not add new insights and only makes the characterisation of the equilibrium less traceable.

Moreover, the worst-case variance $\bar{\sigma}^i_{\max}$ is not relevant for the participation decision, but it determines the optimal supply together with $\bar{\varepsilon}^i_{\max}$ once an AA lender participates.

Third, IBM participation and the volume of liquidity supplied depend on the banking type and lender type. Specifying the supply function for relationship and transactional banking is not clear-cut. The type of lender plays a role. When banks are risk averse, the non-participation area is larger for relationship banking. With this type of banking lenders require a higher interbank rate because relationship banking has a higher cost (see equation 10). On the other hand, for AA lenders, the non-participation zone depends on how the costs $\bar{\varepsilon}^{Ti}_{\max}$ and $\bar{\varepsilon}^{Si}_{\max} + c^i$ relate. When $\bar{\varepsilon}^{Ti}_{\max}$ is smaller than $\bar{\varepsilon}^{Si}_{\max} + c^i$, the non-participation zone is higher with relationships, i.e. AA banks engaging in transactional banking are willing to supply liquidity at lower rates. Vice versa, when $\bar{\varepsilon}^{Ti}_{\max}$ is higher than $\bar{\varepsilon}^{Si}_{\max} + c^i$, participation with relationship banking starts at lower rates. The type of lender determines participation because RA lenders are willing to provide liquidity to the IBM at lower rates due to differences in beliefs[20]. Hence, for an increasing interbank rate, there is a range where no one lends followed by a region where only RA lenders are sufficiently compensated, and finally, for the highest rates, where AA lenders are sufficiently compensated.

Corollary 1 *The willingness to participate in the IBM is influenced by the banking type and lender's type. Relationship banking does not improve the willingness of RA lenders to provide liquidity to the IBM, but for ambiguity lenders it does when $c^i < \bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max}$ or when the gains in perceived ambiguity about the mean exceed the relationship costs. For RA lenders, the non-participation zone is larger for transactional banking than for relationship banking, i.e. when $R^i \in]1+\hat{\varepsilon}^i, 1+\hat{\varepsilon}^i + c^i]$ participation exists only via transactional banking. Further, RA lenders are, compared to AA lenders, willing to provide liquidity to the IBM for a broader set of market conditions: in the region $R^i \in]1+\hat{\varepsilon}^i + I^i c^i, 1+\bar{\varepsilon}^i_{\max} + I^i c^i]$, there exists participation by RA lenders whereas AA ones do not participate.*

Fourth, the lenders' supply differs for the type of banking and lender. For relationship banking, RA lenders provide less liquidity to the IBM because they face relationship costs only and are not influenced by the gains a relationship has on ambiguity. The impact of the type of lending depends for an AA lender on the gap between $\bar{\varepsilon}^{Ti}_{\max}$ and $\bar{\varepsilon}^{Si}_{\max} + c^i$, and on the gap between $\bar{\sigma}^{Ti}_{\max}$ and $\bar{\sigma}^{Si}_{\max}$. One can see that with relationship banking the supply is always larger if $\bar{\varepsilon}^{Ti}_{\max}$ is larger than $\bar{\varepsilon}^{Si}_{\max} + c^i$. By contrast, if $\bar{\varepsilon}^{Ti}_{\max}$ is smaller than $\bar{\varepsilon}^{Si}_{\max} + c^i$, the amount of liquidity supplied is now also subject to the gap between $\bar{\varepsilon}^{Ti}_{\max}$ and $\bar{\varepsilon}^{Si}_{\max} + c^i$ compared to the gap between $\bar{\sigma}^{Ti}_{\max}$ and $\bar{\sigma}^{Si}_{\max}$. Moreover, when $\bar{\varepsilon}^{Si}_{\max} + c^i > \bar{\varepsilon}^{Ti}_{\max}$, supply is larger with relationship banking only if $\bar{\sigma}^{Si}_{\max}$ is sufficiently slower than $\bar{\sigma}^{Ti}_{\max}$. Furthermore, compared to AA lenders, RA lenders provide more liquidity to the IBM because they have precise priors about the participation costs[21].

[20] Differences in beliefs concerning the participation costs result in a different participation zone for a lender. Compare the region of non-participation for AA and RA lenders: $R^i \leq 1 + \bar{\varepsilon}^i_{\max} + I^i c^i$ (Equation 9) and $R^i \leq 1 + \hat{\varepsilon}^i + I^i c^i$ (Equation 6) respectively. Also note that $\hat{\varepsilon}^i < \bar{\varepsilon}^i_{\max}$.

[21] Since $\hat{\sigma}^i < \bar{\sigma}^i_{\max}$ and $\hat{\varepsilon}^i < \bar{\varepsilon}^i_{\max}$ it can be shown that RA lenders are willing to supply more if one compares Equation 9 and Equation 6, which is the IBM supply by AA and RA lenders, respectively.

Corollary 2 *The amount of IBM supply is influenced by the banking type and lender type. Relationship banking does not improve the liquidity supplied by RA lenders; supply is higher for transactional banking, i.e.,* $\frac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i} > \frac{R^i - \hat{\varepsilon}^i - 1 - c^i}{\hat{\sigma}^i}$. *Relationship banking improves IBM liquidity always when the gains in perceived ambiguity about the mean cover the relationship costs, i.e., if* $c^i < \bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max}$. *If not, when* $c^i > \bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max}$, *relationship banking improves IBM liquidity only when the gains in perceived ambiguity about the variance,* $\bar{\sigma}^{Ti}_{\max} - \bar{\sigma}^{Si}_{\max}$, *are large compared to the costs* $\bar{\varepsilon}^{Si}_{\max} - \bar{\varepsilon}^{Ti}_{\max} + c^i$. *Further, RA lenders provide generally more liquidity in the IBM than AA lenders (conditional on participating and for a given interbank rate).*

Fifth, in a world where relationships are free and no relationship costs c^i are incurred, the interbank rate for a partial participation equilibrium (PPE), the same for transactional and relationship banking, equals $\frac{\hat{\sigma}^i \bar{m}^i_B}{1-\mu} + \hat{\varepsilon}^i + 1$. The rate for relationship banking with a full participation equilibrium (FPE) however, equals:

$$\frac{\hat{\sigma}^i \bar{\sigma}^{Si}_{\max} \bar{m}^i_B + \mu \hat{\sigma}^i (\varepsilon^{Si}_{\max} + 1) + (1-\mu)\bar{\sigma}^{Si}_{\max}(\hat{\varepsilon}^i + 1)}{\mu \hat{\sigma}^i + (1-\mu)\bar{\sigma}^{Si}_{\max}}$$

if the lending banks do not incur any relationship costs. This rate does differ from the one with transactional banking, which is due to the beneficial effect of a relationship on the confidence of the lender, and hence, the perceived worst case.

4 Interbank Market Equilibrium and Discussion

4.1 Interbank market clearing

For the equilibrium to hold, two conditions should be satisfied. The first condition, the market-clearing condition, states that the per capita liquidity supply should equal the borrower's liquidity needs, and this applies for every IBM loan. This condition is written as:

$$\mu x^{i*}_{AA}(R^i) + (1-\mu) x^{i*}_{RA}(R^i) = \bar{m}^i_B \tag{12}$$

where $\bar{m}^i_B$ represents the borrower's demand for an IBM loan i, and $x^{i*}_{AA}(R^i)$ and $x^{i*}_{RA}(R^i)$ are the supply functions found in equation (11) and equation (10), respectively. The second equilibrium condition selects, for every loan, the banking type by which borrowers obtain a loan at the lowest equilibrium rate. Borrowers choose relationship banking if this banking type allows them to obtain the necessary funds at a better, i.e. lower, interbank rate than with transactional banking.

Solving for the equilibrium rate is done in several steps. First, for each individual loan the equilibrium rate is determined for transactional and relationship banking. Further, for each type of banking, either full or partial participation arises. While the former stipulates participation by RA lenders, the latter stipulates participation by all lending banks. The banking type and lender type determine the region of participation (see equations 11 and 10). For more extreme values of the worst case, the equilibrium outcome is more likely one with partial participation; otherwise, for a more moderate worst case, the chance of a full participation equilibrium (FPE) is higher.

Proposition (1) shows the participation conditions for transactional and relationship banking, which is explained in more detail in Section 4.1.1 and Section 4.1.2, respectively.

Proposition 1 *For each type of banking, there is a unique equilibrium for loan i in the interbank market.*

- *With transactional banking, the market-clearing rate is either*

1. partial participation equilibrium if $\frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 \leq \bar{\varepsilon}_{\max}^{Ti} + 1$, *where in the equilibrium* $x_{AA}^{*i} = 0, x_{RA}^{*i} = \frac{\bar{m}_B^i}{1-\mu}$, *and* $R_{PPE}^{Ti} = \frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1$ *is the market clearing rate, or*

2 full participation equilibrium if $\frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 > \bar{\varepsilon}_{\max}^{Ti} + 1$, *where in the equilibrium both* $x_{AA}^{*i} = 0, x_{RA}^{*i} > 0$, *and* $R_{FPE}^{Ti} = \frac{\hat{\sigma}^i \bar{\sigma}_{\max}^{Ti} \bar{m}_B^i + \mu \hat{\sigma}^i (\bar{\varepsilon}_{\max}^{Ti} + 1) + (1-\mu) \bar{\sigma}_{\max}^{Ti} (\hat{\varepsilon}^i + 1)}{\mu \hat{\sigma}^i + (1-\mu) \bar{\sigma}_{\max}^{Ti}}$ *is the market clearing rate.*

- *With relationship banking, the market-clearing rate is either*

1. partial participation equilibrium if $\frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 \leq \bar{\varepsilon}_{\max}^{Si} + 1$, *where in the equilibrium* $x_{AA}^{*i} = 0, x_{RA}^{*i} = \frac{\bar{m}_B^i}{1-\mu}$, *and* $R_{PPE}^{Si} = \frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 + c^i$ *is the market clearing rate, or*

2. full participation equilibrium if $\frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 > \bar{\varepsilon}_{\max}^{Si} + 1$, *where in the equilibrium both* $x_{AA}^{*i} = 0, x_{RA}^{*i} > 0$, *and* $R_{FPE}^{Si} = \frac{\hat{\sigma}^i \bar{\sigma}_{\max}^{Si} \bar{m}_B^i + \mu \hat{\sigma}^i (\bar{\varepsilon}_{\max}^{Si} + 1 + c^i) + (1-\mu) \bar{\sigma}_{\max}^{Si} (\hat{\varepsilon}^i + 1 + c^i)}{\mu \hat{\sigma}^i + (1-\mu) \bar{\sigma}_{\max}^{Si}}$ *is the market clearing rate.*

The willingness to participate in the IBM depends on the banking type and lender type. With transactional banking, there is a lower non-participation region, and RA lenders already provide liquidity to the IBM for lower interbank rates. For rates where this type of lender is willing to engage in both transaction and relationship banking, the supply is always higher with transactional banking.

Note also that a comparison between the two types of lender shows that the liquidity provision in the IBM occurs for a broader set of interbank rate values with RA lenders. Hence, the non-participation zone is smaller with a population of RA lenders compared to one with AA lenders. Also, AA lenders supply less than the RA ones when participating.

Borrowers want to fulfil their liquidity needs at the lowest possible rate and, accordingly, choose the type of banking. Proposition (2) stipulates the borrower's choice.

Proposition 2 *The equilibrium rate for loan i is*

1. R^{Ti}_{PPE} *if* $\dfrac{\hat{\sigma}^i \bar{m}^i_B}{1-\mu} + \hat{\varepsilon}^i \leq \bar{\varepsilon}^{Si}_{\max}$
2. $\min[R^{Ti}_{PPE}; R^{Si}_{FPE}]$ *if* $\bar{\varepsilon}^{Si}_{\max} < \dfrac{\hat{\sigma}^i \bar{m}^i_B}{1-\mu} + \hat{\varepsilon}^i \leq \bar{\varepsilon}^{Ti}_{\max}$
3. $\min[R^{Ti}_{FPE}; R^{Si}_{FPE}]$ *if* $\bar{\varepsilon}^{Si}_{\max} < \bar{\varepsilon}^{Ti}_{\max} < \dfrac{\hat{\sigma}^i \bar{m}^i_B}{1-\mu} + \hat{\varepsilon}^i$

Borrowers prefer relationship banking if $R^{Si} \leq R^{Ti}$, otherwise they choose transactional banking. The careful reader should note that when a partial participation equilibrium (PPE) prevails with relationship banking, it implies a PPE also for transactional banking. With partial participation, no borrower will opt for relationship banking because it is more expensive: $R^{Si}_{PPE} > R^{Ti}_{PPE}$. With full participation, the borrower's choice depends on the parameter values of the perceived ambiguity with relationships and its costs. So, in equilibrium borrowers engage in relationship banking only if both types of lenders participate, otherwise, transactional banking occurs[22].

4.1.1 *Transactional banking*

Consider for an individual loan i the equilibrium rate for transactional banking. The unique equilibrium is either one with full participation, i.e. both types of lender participate, or one with partial participation because AA lenders are not willing to lend because of insufficient compensation. For an interbank rate between $1 + \hat{\varepsilon}^i$ and $1 + \bar{\varepsilon}^{Ti}_{\max}$ a partial participation equilibrium (PPE) exists because only RA lenders are willing to provide liquidity to the IBM. A FPE exists for an equilibrium IBM rate that motivates both types of lenders to participate in the IBM, meaning that the rate is sufficiently high to compensate for the worst-case participation costs. This situation arises when the rate is above $1 + \bar{\varepsilon}^{Ti}_{\max}$. For transactional banking, the rate for a PPE is:

$$R^{Ti}_{PPE} = \frac{\hat{\sigma}^i \bar{m}^i_B}{1-\mu} + \hat{\varepsilon}^i + 1. \tag{13}$$

The market-clearing rate R^{Ti}_{PPE} represents the interbank rate that prevails for transactional banking when $1 + \hat{\varepsilon}^i \leq R^{Ti}_{PPE} \leq 1 + \bar{\varepsilon}^{Ti}_{\max}$ or when only RA lenders are participating[23]. When full participation prevails, the market-clearing rate for transactional banking R^{Ti}_{FPE} is as follows:

[22] The economic environment is *i*) one with transactional banking where not all lending banks participate, *ii*) one with either transactional or relationship banking, but in case of the former, only RA loan officers provide IBM loans, whereas with the latter, full participation occurs, and *iii*) one with either transactional or relationship banking where every lending bank participates. Hence, the equilibrium outcome is either one with transactional banking or one where the borrower's choice depends on the parameters of the interbank rate, which is further discussed in Section 5. In the former, transactional banking arises because the beneficial effect of relationships is not at play. Moreover, relationship banking is an equilibrium outcome only when all the lending banks are supplying IBM liquidity, and then, the choice between banking type is relevant.

[23] When the whole population is risk averse, $1-\mu = 1$, the equilibrium rate is as follows: $R^{Ti}_{PPE} = \hat{\sigma}^i \bar{m}^i_B + \hat{\varepsilon}^i + 1$.

(14) $$R_{FPE}^{Ti} = \frac{\hat{\sigma}^i \bar{\sigma}_{\max}^{Ti} \bar{m}_B^i + \mu \hat{\sigma}^i (\bar{\varepsilon}_{\max}^{Ti} + 1) + (1-\mu) \bar{\sigma}_{\max}^{Ti} (\hat{\varepsilon}^i + 1)}{\mu \hat{\sigma}^i + (1-\mu) \bar{\sigma}_{\max}^{Ti}}$$

and represents the interbank rate for transactional banking when both AA and RA lenders participate. So, this rate applies when $1 + \bar{\varepsilon}_{\max}^{Ti} \leq R_{PPE}^{Ti}$. For IBM lending via a transaction, a non-participation zone exists when neither type of lender participates and occurs when interbank rates are below $1 + \hat{\varepsilon}^i$. Intuitively, this emerges when a lender expects his earning to be lower with IBM lending than with investing in the risk-free investment[24]. A situation with participation through a transaction by only AA lenders is possible, but then the whole population should solely include such AA lenders. The market-clearing rate for transactional banking now equals $\bar{\sigma}_{\max}^{Ti} \bar{m}_B^i + \bar{\varepsilon}_{\max}^{Ti} + 1$. On the other hand, if the population includes only RA lenders, the equilibrium rate is $\hat{\sigma}^i \bar{m}_B^i + \hat{\varepsilon}^i + 1$.

4.1.2 *Relationship banking*

Similar to the previous section, a unique equilibrium for relationship banking is found: either one where everyone participates or one where only the RA lenders participate in the IBM. With relationship banking, however, there is a cost for building relationships, i.e., relationship costs c^i. These costs have an impact on a lending bank's return. For increases in costs, the return on loan *i* drops. Lenders need to be compensated sufficiently for providing liquidity to the IBM. Therefore, these costs c^i have an impact on the interbank rate set by a lender. The rate for partial participation with relationship banking is:

(15) $$R_{PPE}^{Si} = \frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i + 1 + c^i$$

and the market-clearing rate for full participation with relationship banking is

(16) $$R_{FPE}^{Si} = \frac{\hat{\sigma}^i \bar{\sigma}_{\max}^{Si} \bar{m}_B^i + \mu \hat{\sigma}^i (\bar{\varepsilon}_{\max}^{Si} + 1 + c^i) + (1-\mu) \bar{\sigma}_{\max}^{Si} (\hat{\varepsilon}^i + 1 + c^i)}{\mu \hat{\sigma}^i + (1-\mu) \bar{\sigma}_{\max}^{Si}}$$

The former applies when AA lenders do not provide liquidity to the IBM through a relationship, and applies for a rate between $1 + \hat{\varepsilon}^i + c^i$ and $1 + \bar{\varepsilon}_{\max}^{Si} + c^i$. Once the interbank rate is high enough and exceeds $1 + \bar{\varepsilon}_{\max}^{Si} + c^i$, both AA and RA lenders participate in relationship banking. A situation with solely AA lenders participating through a relationship, i.e. $\mu = 1$, may arise when the whole lending population is ambiguity averse. When participating, AA lenders then set an equilibrium rate for relationship banking equal to $\bar{\sigma}_{\max}^{Si} \bar{m}_B^i + \bar{\varepsilon}_{\max}^{Si} + 1$. On the other hand, when the whole population is risk averse, the equilibrium rate equals $\hat{\sigma}^i \bar{m}_B^i + \hat{\varepsilon}^i + 1 + c^i$ for $\mu = 0$. With relationships, the

[24] With transactional banking, the non-participation zone for AA lenders exists for interbank rate values below $1 + \bar{\varepsilon}_{\max}^{Ti}$, while for RA lenders this zone arises for values below $1 + \hat{\varepsilon}^i$ (see equations 11 and 10).

non-participation area is for rates below $1 + \hat{\varepsilon}^i + c^i$. The equilibrium is again one with either full or partial participation.

5 Discussion of the Equilibrium and Empirical Predictions

The equilibrium rate that applies for the IBM depends on the economic environment and, more specifically, how $\frac{\hat{\sigma}^i \bar{m}_B^i}{1-\mu} + \hat{\varepsilon}^i, c^i, \bar{\varepsilon}_{\max}^{Si}$ and $\bar{\varepsilon}_{\max}^{Ti}$ relate (see Proposition (2)). Further, the equilibrium outcome reflects a borrower's choice where the lowest possible interbank rate is preferred. This rate, and consequently the equilibrium outcome, is sensitive to changes in population composition, perceived ambiguity, relationship costs, risk, and borrower's demand.

Intuitively, the paper's results reveal a number of interesting features of the IBM. In particular, establishing a relationship is a way of mitigating liquidity shortages if it allows the lender to provide a loan at a favourable interbank rate[25]. This occurs when a lender's improved confidence exceeds the costs of relationship banking[26]. The main results are summarised as follows. A first finding concerns the improved confidence, which is measured in terms of either ambiguity about the mean or ambiguity about the variance, and how this needs to be compared to the costs. Moreover, the benefits of establishing a relationship prevail when many lenders are hit by a loss in confidence, meaning relationship banking exists only when lenders are ambiguity averse. Furthermore, engaging in a relationship is more likely for larger fractions of AA lenders[27]. Second, relationship banking exists, and stimulates IBM liquidity, for large improvements in ambiguity about the mean – compared to transactional banking. On the other hand, for a limiting impact on the ambiguity on the mean, relationships might still arise, but only for relatively extensive limits to the ambiguity about the variance – compared to the small ones in ambiguity about the mean. This result suggests that relationships are particularly interesting in an ambiguous environment like the 2007-2008 crisis and the current crisis in Russia, among others. One might ask when these improvements in perceived ambiguity, in terms of either mean or variance, are large. This crucially depends on the environment's level of ambiguity. For high levels of ambiguity, perceived ambiguity should be limited more when employing relationship banking, which explains that this type of banking allows for better access to IBM funding but does not guarantee it[28].

[25] Affinito (2012), for instance, states that relationships allow lenders to provide liquidity during times of crisis. Afonso, Kovner and Schoar (2013) highlight that a borrower acquires a more favourable rate with relationship banking.

[26] The importance of improved confidence compared to relationship costs can be found in, among others, the paper by Blasques, Bräuning and Lelyveld (2015). This work indicates that the relationship costs significantly reduce uncertainty about a counterparty's risk profile.

[27] For instance, the work by Bräuning and Fecht (2017), Flannery (1996), and Temizsoy *et al.* (2015) show that relationships improve confidence by helping to insure mutual trust and by overcoming informational problems.

[28] Cocco, Gomes and Martins (2009), for instance, find that relationships are an important determinant of a bank's ability to access to funding via the IBM and of the IBM's liquidity level.

Third, relationship banking arises when the borrower is large, complex or distressed[29] due to decreasing relationship costs[30].

5.1 Lenders' population composition

The equilibrium effect of the lenders' population composition is stated in Proposition (3) and explained more elaborately below.

Proposition 3 *Establishing relationship banking in equilibrium is more likely for a μ close to one – a population with nearly all AA lenders – than for low values of μ – a population with nearly no AA lenders: the interbank rate increases slower with relationship banking.*

1. For μ close to zero or a population with nearly all RA lenders, transactional banking is an equilibrium outcome. The corresponding interbank rate is equal to $1 + \hat{\varepsilon}^i$.

2. For μ close to one or a population with nearly all AA lenders, relationship banking is an equilibrium outcome when i) the relationship benefits outweigh the costs, i.e. $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} > c^i$, or (ii) the imagined worst case is low for relationship banking, i.e. when the gap $\bar{\sigma}^{Ti}_{\max} - \bar{\sigma}^{Si}_{\max}$ is large compared to $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} < c^i$.

3. For intermediate levels of μ, relationship banking can also be an equilibrium outcome and it is determined by the relationship benefits and costs, and the level of perceived ambiguity.

The above proposition deals with the influence of the population composition on the equilibrium outcome. The population considers two types of lenders with CARA-normal preferences that provide liquidity to the IBM: a proportion μ of AA lenders and a proportion $1 - \mu$ of RA lenders. The interbank rate increases with the proportion of AA lenders in both a partial participation equilibrium (PPE) and full participation one (FPE). While for a PPE the type of banking has no influence on the interbank rate's sensitivity to μ, for a FPE it does. When all or nearly all lenders are risk averse, μ is close to zero, the interbank rate increases with the proportion of AA lenders and this increase is even stronger with relationship banking. In contrast for a large ambiguous population, values of μ close to one, the interbank rate is more sensitive to transactional banking.

When both lender types are represented, relationship banking adds value when AA lenders participate and when the difference in perceived ambiguity is sufficiently high to cover costs related to the relationship. The difference in perceived ambiguity influences the vertical deviation and steepness of the per capita supply curve. The preferred type of banking depends on the gap between the relationship costs and benefits. The importance

[29] The research by Hatzopoulos *et al.* (2016) shows that relationship banking involves large trading volumes, which is motivated by the limited funding possibilities for large banks and large funding needs. On the other hand, Bräuning and Fecht (2017) highlight the importance of borrower's opaqueness for relationship banking.

[30] Remember that relationship costs represent, for instance, the loss when a negatively correlated bank pair is affected by a structural break, which also impairs the liquidity insurance mechanism. Other explanations for these costs are the investments made in screening, negotiating, and monitoring to acquire information about the counterparty and to keep it updated. The relationship costs can be considered to be decreasing due to economies of scale, independent quality validation, too-big-to-fail and too-complex-to-fail factors.

of perceived ambiguity is discussed in the next section. On the other hand, when a PPE materialises borrowers choose optimally transactional banking.

Relationship banking can also add value for $\mu = 1$ through the beneficial effect of ambiguity reduction (that exceeds the costs). In particular, this occurs when participation starts earlier, i.e. at lower rates, compared to transactional banking: for values $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} > c^i$, a borrower prefers relationship banking[31]. Besides, the steepness of the per capita supply curves also determines the type of banking that arises for $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} < c^i$. Although participation starts earlier with transactional banking, supply is higher with relationships from certain interbank rates due to the difference in the imagined worst case[32]. This is also relevant when $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} < c^i$: the per capita supply for a range of rates is higher with relationship banking because of the difference in the imagined worst case.

Alternatively, a relationship in the IBM does not add value when $\mu = 0$ and the population exists in RA lenders. Moreover, borrowers choose transactional banking for relative low values of μ. Although a relationship is costly, it has no impact on a RA lender's beliefs and does not improve the confidence in measuring the borrower's quality correctly. Transactional banking will prevail in equilibrium at an interbank rate equal to $\hat{\sigma}^i \bar{m}^i_B + \hat{\varepsilon}^i + 1$. Due to the relationship costs c, participation in relationship banking arises for values $R^i > 1 + \hat{\varepsilon}^i + c^i$ compared to $R^i > 1 + \hat{\varepsilon}^i$ in transactional banking.

5.2 Perceived ambiguity

How perceived ambiguity determines the equilibrium is summarised in the following proposition and explained below. The perceived ambiguity with relationship banking differs from transactional banking, meaning that a lender's aversion to the induced subjective uncertainty is different. This difference in perceived ambiguity influences the participation decision of AA lenders, the interbank rate, and the amount of liquidity supplied to the IBM. Relationship banking allows banks to limit the perceived ambiguity, which also means that an AA lender's beliefs about the worst-case mean and variance are bounded.

Proposition 4 *The equilibrium outcome is determined by perceived ambiguity along two dimensions:*

1. Relationship banking prevails for large differences in perceived ambiguity about the mean (between the types of banking) through:

- *the participation decision of AA lenders: $\bar{\varepsilon}^{Ti}_{\max} >> \bar{\varepsilon}^{Si}_{\max}$. The extent of the difference needed for relationship banking to prevail grows with the level of ambiguity in the environment.*

[31] For relationship banking, AA lenders participate at interbank rate levels exceeding $1 + \bar{\varepsilon}^{Si}_{\max} + c^i$, whereas transactional banking starts from rates exceeding $1 + \bar{\varepsilon}^{Ti}_{\max}$. Whether participation starts earlier, i.e. at lower rates, with transactional or relationship banking, depends on the parameter values of $\bar{\varepsilon}^{Ti}_{\max}$ and $\bar{\varepsilon}^{Si}_{\max}$. Three possibilities arise: *i*) if $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} = c^i$, then borrowers are indifferent for the type of banking, *ii*) if $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} > c^i$, then borrowers prefer relationship banking, and *iii*) if $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} < c^i$, then transactional banking, not relationship banking, is preferred by borrowers up to a certain interbank rate.

[32] This supply curve is more steep for relationship banking than for transactional banking (i.e. $\bar{\sigma}^{Si}_{\max} < \bar{\sigma}^{Ti}_{\max}$) and the difference in the imagined worst case has an influence on supply curve vertical deviation (i.e., $\bar{\varepsilon}^{Si}_{\max} < \bar{\varepsilon}^{Ti}_{\max}$) and steepness (i.e., $\bar{\sigma}^{Si}_{\max} < \bar{\sigma}^{Ti}_{\max}$).

- *the interbank rate as* $\bar{\varepsilon}_{\max}^{Ti} - \bar{\varepsilon}_{\max}^{Si}$ *increases.*

2. Relationship banking can also prevail for small differences in perceived ambiguity about the mean (between the types of banking), but only when the perceived ambiguity about the variance is relatively large, i.e. when $\bar{\sigma}_{\max}^{Ti} - \bar{\sigma}_{\max}^{Si}$ *grows faster than* $\bar{\varepsilon}_{\max}^{Ti} - \bar{\varepsilon}_{\max}^{Si}$*.*

The difference between $\bar{\varepsilon}_{\max}^{Ti}$ and $\bar{\varepsilon}_{\max}^{Si}$ reflects the difference in perceived ambiguity about the mean for the two types of banking. Relationship banking prevails when $\bar{\varepsilon}_{\max}^{Ti}$ is large relative to $\bar{\varepsilon}_{\max}^{Si}$. If this difference however grows faster than the gap $\bar{\sigma}_{\max}^{Ti} - \bar{\sigma}_{\max}^{Si}$ but remains below c^i, then transactional banking occurs. Also, for increases in ambiguity, participation by AA lenders is discouraged and relationship banking becomes more difficult to attain in equilibrium: a higher difference in perceived ambiguity about the mean for the banking types is needed. Generally, for low levels of ambiguity, full participation is more likely, by which relationship banking is also more likely to be an equilibrium outcome[33].

5.3 Relationship costs

The equilibrium outcome is also characterised by how costly relationships are. This characterisation is stated as follows.

Proposition 5 *For increases in relationship costs* c^i*, due to size or complexity, relationship banking is more likely to be an equilibrium outcome. Banks may economise for higher values of* c^i *and, hence, the costs are disproportionally lower.*

The costs for relationship banking increase in size and complexity but are capped[34], resulting in disproportionally lower costs for the lending bank when a borrower is larger, organised complexly and/or distressed. Thus, relationship banking emerges as the preferred type of banking for those borrowers because its beneficial effect is larger than the corresponding costs. Moreover, lending banks are able to provide a loan with relationship banking at a more appealing rate, which borrowers will prefer to transactional banking. Vice versa, this trade-off between benefits and costs results in the choice of transactional banking for small, simple, and untroubled banks.

[33] For a PPE, the interbank rate demanded by RA lenders is not influenced by changes in ambiguity. On the other hand, for a FPE the rate increases with ambiguity via both $\bar{\varepsilon}_{\max}^{i}$ and $\bar{\sigma}_{\max}^{i}$. Relative to transactional banking, relationship banking is more sensitive to changes in ambiguity (because $\bar{\sigma}_{\max}^{Ti} > \bar{\sigma}_{\max}^{Si}$ and $\bar{\varepsilon}_{\max}^{Ti} > \bar{\varepsilon}_{\max}^{Si}$). For increases in the level of ambiguity, the interbank rate for relationship banking grows faster than with transactional banking, meaning that the beneficial effect of the former becomes less important, and hence, transactional banking will probably emerge for high ambiguity levels. The reasoning for this is that relationships are more sensitive to changes in ambiguity.

[34] One can think about these capped relationship costs as a result of economies of scale, independent quality validation, too-big-to-fail and too-complex-to-fail factors. Affinito (2012) for instance, states that substantial borrowers are considered to be safe. Moreover, these large and complex borrowers have high and volatile liquidity needs, but their bank portfolio is safe, successful and multifaceted. Moreover, the author highlights the importance of the connectedness for relationships. In related study, Finger and Lux (2017) show that larger and core banks are less likely to default because of regulatory intervention. Also Langfield *et al.* (2014) point to economies of scale for diversified activities.

5.4 Risk

The findings about the impact of risk on the equilibrium are summarised in the following proposition.

Proposition 6 *For increases in the level of risk, the equilibrium outcome only changes with full participation: relationship banking is more likely to be the equilibrium outcome because transactional banking is more sensitive to the level of risk because* $\bar{\sigma}^{Ti}_{\max} > \bar{\sigma}^{Si}_{\max}$ *and* $\bar{\varepsilon}^{Ti}_{\max} > \bar{\varepsilon}^{Si}_{\max}$.

Relationship banking does not occur in a PPE, and the prevailing banking type will not change due to changes in the risk parameters for a PPE: relationship banking only constrains the perceived ambiguity. Nevertheless, a RA lender's decision to participate is influenced by the level of risk about the mean, i.e. $R^i > 1 + \hat{\varepsilon}^i + I^i c^i$, so there is a wider range of interbank rate values where no participation occurs. Moreover, for RA participation, the amount of liquidity supplied to the IBM decreases with risk about the mean and risk about the variance.

For a FPE, risk parameters have a larger impact for transactional banking: the interbank rate grows faster with the level of the risk parameters (as opposed to relationship banking). Hence, relationship banking is the most probable equilibrium outcome. Also, the interbank rate's sensitivity to risk is larger for a PPE than for a FPE, resulting from the type of lenders that participate. Therefore, it may occur that a FPE becomes the more likely outcome, and likewise, relationship banking becomes more likely.

5.5 Borrower's demand

A borrower's demand has an influence on the equilibrium interbank rate, which increases with borrowing demand as stated in Proposition (7).

Proposition 7 *For increases in borrower's demand, the equilibrium outcome only changes with full participation: relationship banking is more likely to prevail in equilibrium with increases in the borrower's demand – because the corresponding interbank rate is less sensitive than with transactional banking – and when the risk about variances is relatively large, i.e., for* $\hat{\sigma}^i > 1 - \mu$ *For a relatively small risk about variances, i.e., for* $\hat{\sigma}^i < 1 - \mu$*, transactional banking will prevail in a FPE as relationship banking has a higher sensitivity to a changing borrowers' demand.*

Generally, the interbank rate changes with borrower demand more strongly in a FPE compared to a PPE. The equilibrium outcome only changes when the difference between transactional and relationship banking is relevant, i.e. only in a FPE, and depends on the parameter value of risk about the variance $\hat{\sigma}^i$ and the proportion of RA lenders in the lending population. When $\hat{\sigma}^i > 1 - \mu$ – risk is relatively large – lenders care about variances and the interbank rate with transactional banking grows more with borrower demand than the rate with relationship banking. Hence, relationship banking is the most likely outcome.

5.6 Empirical predictions

Supply in the IBM increases nonlinearly with the interbank rate and shows that lending banks' willingness to lend increases with the interbank rate. For increasing interbank rates, participation suddenly increases as another type of lender is sufficiently compensated. For low rates, below $1 + \hat{\varepsilon}^i$, no lending bank is willing to provide liquidity to the IBM, and participation by RA lenders starts for interbank rates larger than $1 + \varepsilon^i$ and transactional banking with a PPE occurs. Relationship banking does not exist as a borrower's choice is driven by the lowest interbank rate.

As long as solely RA lenders are participating, for the region $R^i \in [1 + \hat{\varepsilon}^i; 1 + I^i c^i + \bar{\varepsilon}^i_{\max}]$ the supply curve is:

$$x^i_{RA} = \left\{ \frac{R^i - \hat{\varepsilon}^i - 1}{\hat{\sigma}^i}, \right.$$

which shows that transactional banking prevails. The RA lenders provide liquidity sufficient to match borrowers' demand, $\bar{m}^i_B = (1-\mu) x^{i*}_{RA}(R^i)$.

Once the interbank rate reaches a level above $1 + I^i c^i + \bar{\varepsilon}^i_{\max}$, AA lenders are motivated to participate in the IBM. With full participation, the supply function is flatter because AA lenders need higher compensation to lend a certain amount. Whether AA lenders are participating first, i.e. for lower interbank rates, with transactional or relationship banking depends on the parameter values. Moreover, depending on whether $\bar{\varepsilon}^{Ti}_{\max}$ is larger or smaller than $\bar{\varepsilon}^{Si}_{\max} + c^i$, full participation starts with transactional banking or relationship banking (see Section 5.2). If it starts with transactional banking, it might only be to a certain interbank rate because relationship banking might become appealing. This is due to the steepness of the per capita supply curve for relationship banking.

Supply by an AA lender for loan i is agreed upon through either transactional or relationship banking:

$$x^i_{AA} = \frac{R^i - I^i c^i - \bar{\varepsilon}^i_{\max} - 1}{\bar{\sigma}^i_{\max}} = \begin{cases} \dfrac{R^i - \bar{\varepsilon}^{Ti}_{\max} - 1}{\bar{\sigma}^{Ti}_{\max}} \; if \; R^i - \bar{\varepsilon}^{Ti}_{\max} > 1 \\ \dfrac{R^i - c^i - \bar{\varepsilon}^{Si}_{\max} - 1}{\bar{\sigma}^{Si}_{\max}} \; if \; R^i - c^i - \bar{\varepsilon}^{Si}_{\max} > 1 \end{cases}$$

Relationship banking occurs when it adds value for both of the counterparties. A lender needs higher compensation and a borrower prefers to pay as little as possible for acquiring funds. Therefore, the benefits of a relationship, such as confidence and mutual trust, should limit the ambiguity level such that it is worthwhile to make the investment. Each lending bank has an idea about this advantage and takes it into account when setting the interbank rate. Borrowers then decide whether it is in their best interest to engage in relationship banking[35].

The type of banking that originates is typical for a certain type of borrower. For instance, large, complex borrowing banks, are highly interested in relationship banking because of the savings in the interbank rate. In contrast, small, simple banks prefer transactional banking because they do not have a size advantage for the interbank rate.

From another view, the economic environment may influence the type of banking that is preferred. In times of financial turmoil, or, more generally, in a less familiar, more ambiguous environment, the model predicts that IBM participation drops and only a certain group of banks are willing to provide liquidity. This group is more confident in forming odds about counterparties and they remain active in lending liquidity. Relationship banking is therefore more difficult to attain unless large gains in confidence occur. By contrast, when only risk is at play, relationship banking is most likely to be preferred because with transactional banking, changes in the interbank rate are stronger and in normal times all banks are participating and providing liquidity to the IBM.

In order to truly understand participation in response to changes in economic conditions, it is important to learn more about attitudes towards ambiguity and risk. In financial markets, people are typically not told the odds and this becomes crucial in times of crisis especially. So, for a population where nearly all lenders care about ambiguity, relationship banking occurs for relatively low costs or when perceived ambiguity is significantly limited compared to transactional banking. In contrast, we show that if people would not care about ambiguity, and only care about risk, transactional banking is preferred.

6 Conclusions

This paper shows how relationship banking is potentially beneficial when reducing ambiguity in the interbank market (IBM). Confidence about a counterparty's credit quality influences a lender's decision to lend, and in turn, might distort the IBM efficiency. Therefore, the paper includes a discussion of how microstructure features are at play for inducing ambiguity. The distortion in IBM efficiency occurs when the lender is highly affected by ambiguity, and more specifically, by the perceived worst case. This leads certain leaders to be unwilling to provide liquidity to the IBM, and hence, there is no full participation in the IBM. Although relationship banking is costly, it can persuade ambiguity-averse lenders to participate in IBM lending. It is shown that relationship banking is preferred by borrowers for certain parameter values or more intuitively, when there is a certain gain in perceived ambiguity: relationship banking prevails when much can be gained. Therefore, relationships are especially relevant in crisis times as much confidence can be gained. Further, inducing participation can then be done by influencing the interbank rate via the level of perceived ambiguity, the number of ambiguity-averse

[35] Section 5.1 shows that the perceived ambiguity and relationship costs are relevant for the equilibrium outcome. When the beneficial effects of relationship banking are large enough, i.e. $\bar{\varepsilon}^{Ti}_{\max} - \bar{\varepsilon}^{Si}_{\max} > c^i$, borrowers prefer relationship banking; otherwise, they prefer transactional banking.

lenders, the costs related to relationship building, the level of risk, and the demanded liquidity by borrowers.

The interbank market works quite differently from the bank-firm financial system market, and relationships are an especially interesting aspect to address. Borrowers prefer the lowest possible interbank rate and choose the type of banking (transactional or relationship) accordingly. The type of banking affects IBM liquidity and its evolution, through the beneficial effect on the interbank rate and volume. As argued in this paper, relationship banking can induce lenders to engage in lending that would not occur otherwise. For full participation or a fully efficient IBM, a lender has incentives to charge a lower interbank rate to the borrowing bank with which it has a relationship. It is also important to note that this paper demonstrates that in crisis times, the interbank rate evolves faster for relationship banking, whereas in normal times, it increases more slowly. Establishing relationships is important in crisis times, and naturally, relationship banking becomes more favourable when much can be gained by it. These gains especially relate to changing the lenders' perception about a counterparty's creditworthiness, i.e. their confidence in measuring the credit risk profile correctly. Moreover, the above analysis suggests that relationship banking occurs when borrowers are large, complex and/or distressed, whereas transactional banking is more likely to arise when the opposite is true.

As the banking type determines the IBM efficiency in crisis and non-crisis times, it will have an impact on the effectiveness of a central bank's response[36]. The Euro system regulators, along with regulators in many other systems around the world, responded to the 2007-2008 financial crisis with policy interventions that tried to stimulate IBM participation, via for instance, the broadening of eligible collateral. Moreover, central banks intermediated for large parts of the global financial system that were affected during the heat of the chaos. Using a framework with ambiguity and risk aversion, the model in this paper explores how a borrowing bank then chooses between transactional and relationship banking. In an ambiguous environment, liquidity may be transferred efficiently via relationship banking as opposed to transactional banking. Relationship banking allows a bank to form priors more precisely and this induces participation as a consequence. In particular, a bank may be less doubtful about the credit quality of a borrower and its ability to form a unique probability distribution. To put it differently, relationships are effective in reducing ambiguity and their effectiveness is larger when much can be gained; when engaging in a relationship banks need to compare the worst case outcome imagined against the costs involved. On the other hand, with transactional banking IBM participation may be limited.

The analysis in this paper is relevant for regulators and central banks – policy and regulation should react efficiently in terms of microstructure features – but it is also relevant for lending banks with regards to portfolio choice. Besides considering the links between banks in the IBM, regulators should account for how preferentially connected banks are, and therefore, determine the optimal level of relationships. This might be a point of discussion for Europe when considering the current situation with the quantita-

[36] Whereas Allen and Gale (2000) analyse the structure, efficiency, and resilience of the IBM, Renard and Wuyts (2014) analyse policy intervention in the IBM in relation to ambiguity.

tive easing program and low IBM participation. On the other hand, for a lending bank, this paper can be extended such that many products with varying degrees of quality are included. It is also relevant for risk-averse lenders who may feel unqualified in assessing the counterparty's creditworthiness for a portfolio of many products.

In light of declining relationship costs over time, this paper is a first step to a dynamic setting where relationships in the long run are non-random. Moreover, situations arise in various disciplines where costly services are provided and an agent's choice is influenced by ambiguity and/or risk.

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