Jan De Spiegeleer, Ine Marquet, Wim Schoutens

Data Mining of Contingent Convertible Bonds

(doi: 10.12831/88825)

Journal of Financial Management, Markets and Institutions (ISSN 2282-717X)
Fascicolo 2, giugno-dicembre 2017

Ente di afferenza:
()

Journal of Financial Management Markets and Institutions

Data Mining of Contingent Convertible Bonds

Jan De Spiegeleer
RiskConcile, KU Leuven - Department of Mathematics

Ine Marquet
KU Leuven - Department of Mathematics

Wim Schoutens
KU Leuven - Department of Mathematics

Abstract

This paper develops and applies sophisticated data mining techniques to detect in an early stage potential risks regarding the stability of institutions by making use of the market information of their issued contingent capital instruments. Data mining is an important new topic within the financial world. The detection of observations which are different from the majority, called outliers, can be of interest for market analysts, risk managers, regulators and traders. These exceptions might be caused by extraordinary circumstances that may potentially require extra hedging or can be seen as trading opportunities. They could also give regulators an early warning and signal for potential trouble ahead. In this paper we first explain and apply the new risk measure, called the Value-at-Risk Equivalent Volatility (VEV). The concept was introduced by the European authorities in the new PRIIPs[1] regulation and needs to be implemented for all structured products by January 1st 2018. This risk-measure is an extension of the classical volatility measure by taking into account skewness and kurtosis. This measure however works in a one-dimensional setting. In this paper we apply outlier detection and the VEV concept to CoCo bonds. CoCos are hybrid high yield securities that convert into equity or write down if the issuing financial institution is in a distressed situation. Further we want to detect outliers in the CoCo market taking into account multiple variables such as the CoCo market returns and the underlying equity return. Based on a multiple-dimension distance we can detect CoCos that are outlying compared to previous time periods but also taking into account extreme moves of the market situation. To some extent, CoCos can be seen as derivative instruments with some capital ratio (CET1) as underlying driver. In this perspective, a CoCo market price is just the price of a derivative and hence contains forward looking information or at least the market's anticipated view on the financial health of the institution and the level of the relevant trigger. This paper develops data mining techniques that incorporate such forward looking view by comparing historical data with current CoCo market prices.

Keywords: Contingent Capital; Financial Regulation; Outlier Detection.
JEL Codes: G13; G21; G28; G32.

1 Introduction

Data mining is an important new topic within the financial world. The detection of observations which are different from the majority, called outliers, can be of interest for

Corresponding author: Wim Schoutens, wim.schoutens@kuleuven.be, tel. +32 16322027, KU Leuven - Department of Mathematics, Celestijnenlaan 200B, B-3001 Leuven, Belgium.

[1] Packaged Retail and Insurance-based Investment Products.

ISSN 2282-717X

market analysts, risk managers, regulators and traders. These exceptions might be caused by exceptional circumstances and are requiring extra hedging or can be seen as trading opportunities. They could also give regulators an early warning and signal for potential trouble ahead. In this paper we will investigate the risk of Contingent Convertible (CoCo) bonds. These hybrid high-yield instruments contain an automatically triggered loss-absorption mechanism. These securities convert into equity or write-down if the issuing financial institution is in a distressed situation. Each CoCo bond has specific characteristics and the lack of standardisation proves to be a real challenge. Together with specific characteristics each CoCo also suffers from multiple types of risk. Examples of this are the conversion or write-down risk, coupon cancellation risk, extension risk, etc. These items are discussed in Corcuera *et al.* (2014), Campolongo *et al.* (2017), De Spiegeleer and Schoutens (2012; 2013; 2014), De Spiegeleer *et al.* (2014), Maes and Schoutens (2012). This has made the risk included in CoCos a much discussed point. To some extent, CoCos can be seen as derivative instruments with some capital ratio (CET1) as underlying driver. In De Spiegeleer *et al.* (2017), the concept of implied CET1 volatility was developed in such a context.

Work is conducted firstly in a one-dimensional setting where we explain and apply the new risk measure, called the Value-at-Risk Equivalent Volatility (VEV), to different CoCos. The concept was introduced by the European authorities in the new PRIIPs regulation and needs to be implemented for all structured products by 2018. This risk-measure is an extension of the classical volatility measure by taking into account skewness and kurtosis (European Commission, 2017).

In financial markets, extreme CoCo price movements occur in general when the underlying equity prices move extremely. This relation is clear by construction of the CoCo asset class. However to detect outliers in the CoCo market one should take into account multiple variables such as the CoCo market returns and the underlying equity returns. A robust measure for the autocorrelation is defined to detect outlying behaviour in a multi-dimensional setting. Based on this distance, CoCos can be detected that are outlying compared to previous time periods but also through taking into account extreme moves of the market situation as well.

This paper is organised as follows. In section 2, we explain the construction of Contingent Convertible (CoCo) bonds in order to provide insights on the risk of these products. Subsequently, we measure their risk in terms of the new Value-at-Risk Equivalent Volatility (VEV) measure. With this measure we will also be able to detect the hybrid character of the CoCos. In section 3, we move on to a multivariate analysis. We will use the correlation between the stock market and the CoCo market. The Multivariate Determinant Covariance method is then explained in order to detect outlying CoCos. Section 4 applies the developed techniques to a broad space of currently traded CoCo bonds. Furthermore, we focus on a few particular examples of cases of institutions, namely Deutsche Bank and Banco Popular, that underwent some severe stress recently. Section 5 concludes.

2 Contingent Convertible (CoCo) Bonds

Contingent Convertible (CoCo) bonds are hybrid instruments issued as debt instruments but can be written down or converted to equity in situations of stress. CoCos are high yield instruments that contain an automatic loss-absorption mechanism for the issuer. Depending on the type of CoCo, this loss-absorption can be a full or partial write-down of the notional amount, or consist of a conversion to equity. The trigger of this mechanism is often expressed in terms of the Common Equity Tier 1 (CET1) ratio dropping below a certain predefined level. Next to this, CoCos can be triggered also by supervisors' discretion. The trigger will as such automatically make the investor in CoCos bear part of the losses of the financial institution in stress events (Avdjiev *et al*., 2013; Flannery, 2009).

CoCos have their roots in the financial crisis of 2007-2008. Their purpose is to create extra capital for a distressed bank while keeping it as a going concern. From the first CoCo in 2009, the outstanding CoCo market has grown to over € 150 bn. Due to the loss absorbing function, CoCos count as regulatory capital for the issuer (Basel Committee on Banking Supervision, 2010). A distinction has been made between Additional Tier 1 (AT1) CoCo bonds and Tier 2 (T2) CoCos. AT1 CoCo bonds have a more permanent character given the fact that they are perpetuals. The first call date has to be at least 5 years after the issue date of the bond. A particular property of the coupons distributed by AT1 CoCos is the fact that these coupons can be cancelled. Such a cancellation would not be considered as a default, in contrast with the cancellation of coupon payments on T2 bonds or senior bonds. Furthermore, there is no incentive included for the issuer to pay coupons. The coupons of AT1 CoCo bonds are non-cumulative in a sense that the cancelled coupon payment is lost forever.

Due to the hybrid character and the specific attributes of each CoCo, the risks included in CoCos is often a point of discussion. The time of conversion or write down remains unclear due to the inclusion of the regulator trigger. Furthermore, most pricing models for CoCos take the accounting trigger into account but have no idea of the time that the regulator will trigger these bonds. This lack of knowledge is often translated to ignoring the point of non-viability (McDonald, 2013; Allen and Tang, 2016). This can induce a high risk in modeling the trigger event. Also with an unknown maturity, recovery rate uncertainty and negative convexity contribute to the risk in these instruments. Different approaches have been displayed in the literature to measure multiple risk components. For example the unclearity of the extension risk of CoCos has been investigated in De Spiegeleer and Schoutens (2014). The downward spiral effect of CoCos has been researched in among others De Spiegeleer *et al*. (2014), De Spiegeleer and Schoutens (2013) and De Spiegeleer and Schoutens (2012).

3 Value-at-Risk Equivalent Volatility (VEV)

Financial analyst often refer to volatility as a risk metric expressing the uncertainty in the returns of their products and portfolios. A CoCo typically sits between debt an equity

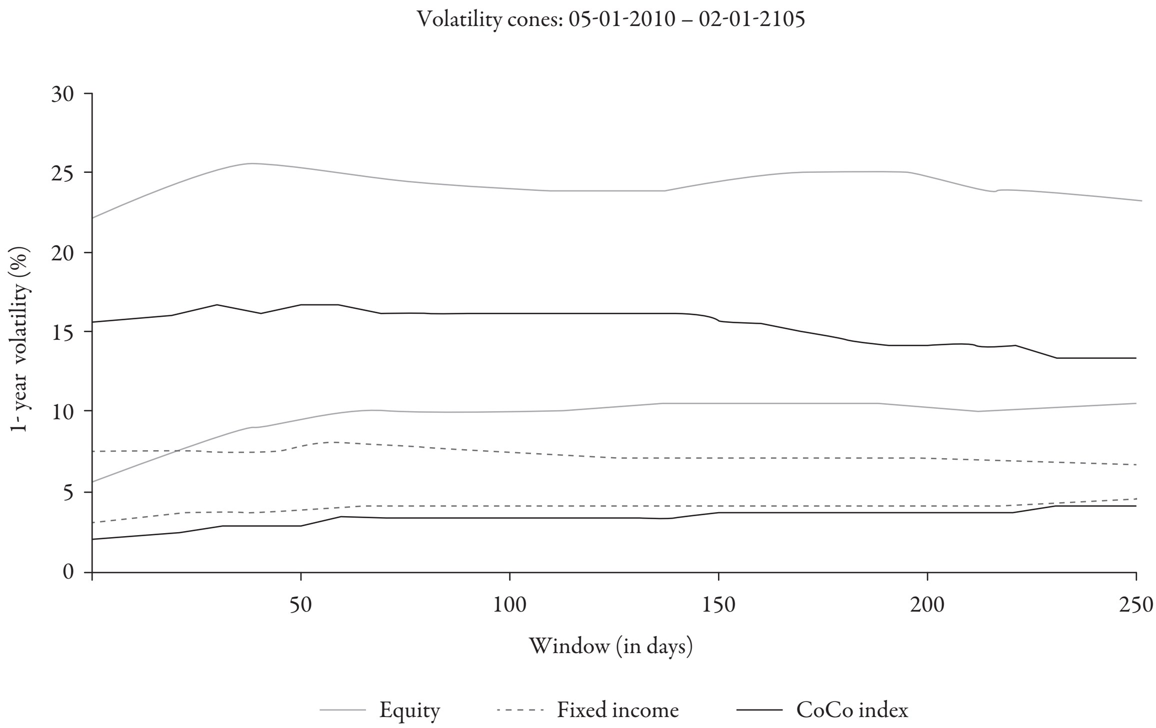


Figure 1: The volatility cone displays the 10% until 90% percentiles of the annualized volatility for multiple window sizes (in days). The cone displays the range of risk for the different asset classes: iShares Core S&P 500 equity ETF, iShares iBoxx Investment Grade fixed income ETF and Credit Suisse CoCo Index.

in terms of volatility. This confirms the hybrid nature of the CoCo bond. Preferreds also exhibit similar volatility. A volatility cone displays historical volatility values for multiple window sizes. The cone is constructed out of 90% (and 10%) upper (resp. lower) bounds for the volatility. These boundaries come closer to each other for longer windows due to the diversification of returns. The risk of the CoCo bond is at an intermediate level in between the cone of the equity and bond (Figure 2).

Regulators want clarity and transparency for the financial instruments offered to investors. Pre-contractually, a retail investor receives for a Packaged Retail and Insurance-based Investment Product (PRIIP) a simple document called Key Information Document (KID), with clear facts and figures on the risks of a particolar financial instrument. The new technical standards classify PRIIPs using a new indicator called the «Summary Risk Indicator» (SRI). This integer number takes values in a range from 1 to 7. Market and credit risk are taken as the major factors of risk that need to be reflected in this indicator, alongside liquidity risk (European Commission, 2017). The determination of market risk relies on the concept of Value-at-Risk Equivalent Volatility (VEV). VEV is calculated based on the Value-at-Risk (VaR) levels of a product and translates this value back to the concepts of volatility.

The Value-at-Risk Equivalent Volatility (VEV) is a new risk measure to evaluate different assets. The VEV denotes the volatility that corresponds with such a VaR loss event. For the PRIIPs, the market risk is measured by the annualised volatility corresponding to the Value-at-Risk (VaR) measured at the 97.5% confidence level over the recommended holding period unless stated otherwise. The VEV formula is a closed form formula.

Under the Black-Scholes model, we know that stock prices are lognormally distributed with drift μ and σ volatility. It follows that the logreturns over a time period T becomes distributed as:

$$r_T \sim N\left[\mu - \frac{\sigma^2}{2}T; \sigma^2 T\right]. \tag{1}$$

For a zero drift, we find the Value-at-Risk (VaR) formula:

$$VaR_{1-\alpha} = -\frac{\sigma^2 T}{2} - \sigma\sqrt{T}\, z_\alpha, \tag{2}$$

with volatility σ, time period T and z_α the α-percentile of a standard normal distribution. For $\alpha = 2.5\%$, the value-at-risk becomes:

$$VaR_{97.5\%} = -\frac{\sigma^2 T}{2} + 1.96\sigma\sqrt{T} \tag{3}$$

A substantial amount of literature has been created on the disadvantages of the normal distribution as a model for financial returns on different markets and periods (see for example Mandelbrot, 1963 and Brinner, 1974). In this perspective the traditional volatility (e.g. used above in the volatility cone) is not a perfect risk measure. Volatility is a symmetric measure by means of treating returns above and below the expected return equally. However, in practice most logreturns turn out to be asymmetrically distributed and have fatter tails, which is referred to in statistics by resp. skewness and kurtosis. These measures can be derived from the central moments, denoted by $M_k = E[(R - E(R))^{k+1}]$ for $k = 1, 2, 3$. The variance, skewness and excess kurtosis of the logreturns become:

$$V = \sigma^2 = M_1 \tag{4}$$

$$S = \frac{M_2}{\sigma^3} \tag{5}$$

$$K = \frac{M_3}{\sigma^4} - 3 \tag{6}$$

Over N trading periods, we have a sample mean of (daily) returns R_i, defined by:

$$\overline{R} = \frac{1}{N}\sum_{i=1}^{N} R_i \tag{7}$$

The variance, skewness and kurtosis of this sample mean become:

$$V_N = \frac{V}{N} \tag{8}$$

$$S_N = \frac{S}{\sqrt{N}} \tag{9}$$

$$K_N = \frac{K}{N} \tag{10}$$

To extend the VaR for non-normal distributed variables, the regulation applies the Cornish-Fisher expansion (Cornish and Fisher, 1938). This expansion estimates quantiles of the non-normal distribution based on the first four moments by:

$$\begin{aligned} VaR_{1-\alpha} &= -\frac{\sigma^2 N}{2} + \sigma\sqrt{N} \\ &\left(z_\alpha + (z_\alpha^2 - 1)\frac{S_N}{6} + (z_\alpha^3 - 3z_\alpha)\frac{K_N}{24} - (2z_\alpha^3 - 5z_\alpha)\frac{S_N^2}{36}\right) \end{aligned} \tag{11}$$

$$\begin{aligned} VaR_{1-\alpha} &= -\frac{\sigma^2 N}{2} + \sigma\sqrt{N} \\ &\left(z_\alpha + (z_\alpha^2 - 1)\frac{S}{6\sqrt{N}} + (z_\alpha^3 - 3z_\alpha)\frac{K}{24N} - (2z_\alpha^3 - 5z_\alpha)\frac{S^2}{36N}\right) \end{aligned} \tag{12}$$

The 1-year VaR Equivalent Volatility (VEV) becomes the volatility in Formula (3) when the VaR is given by Formula (12). This results in:

$$VEV = \left(\sqrt{z_\alpha^2 - 2VaR_{1-\alpha}} + z_\alpha\right)/\sqrt{T}, \tag{13}$$

with T the length of the recommended holding period in years.

Note that the VEV for a normal distribution ($S = 0$, $K = 0$) becomes:

$$VEV = \sigma\sqrt{\frac{N}{T}}$$

If the recommended holding period is equal to N trading days, the length T equals to $N/250$ years. As such, under the normal distribution, we can rescale the daily volatility to a 1-year VEV by multiplying with factor $\sqrt{250}$.

Table 1: Scope of CoCos: Number of CoCos per issuer

Issuer	AT1	T2	Total
UBS	4	5	9
BACR	6	2	8
CS	3	4	7
LLOYDS	6	0	6
ACAFP	5	1	6
SOCGEN	5	0	5
HSBC	5	0	5
RABOBK	3	1	4
DB	4	0	4

3.1 Common pitfalls

The closed form formula (13) can be evaluated for each financial instrument based on the first four moments of its historical returns. However the measure remains a backward looking approach since these moments will be derived from the historical time series. Attention should also be given to the definition of the recommended holding period. A longer recommended holding period reduces the skew and kurtosis parameter. As such, the VEV becomes closer to the volatility. Additionally the application of the Cornish-Fisher expansion requires extra attention. The skewness and kurtosis values in the expansion are estimated parameters and do not coincide with the actual kurtosis and skewness of the instrument. Furthermore the formula is only applicable in a certain range of parameter values which is referred to as the domain of validity.

3.1.1 *Recommended holding period*

There is a major impact on the VEV value by changing the recommended holding period of the PRIIP. The European Supervisory Authorities (incl. ESMA, EIOPA and EBA) point out that a brief description should be given of the reasons for the selection of the recommended holding period and, where present, the required minimum holding period. In addition each KID document has to indicate that the risk of the product if not held to maturity or for the recommended holding period may be significantly higher than the one represented in the report (European Commission, 2017).

In a case study we show the impact on the VEV risk measure due to a change in recommended holding period. The study investigates 103 CoCos consisting of 82 AT1 CoCos and 21 Tier 2 CoCo. In Table 1 we display the number of CoCos issued per bank.

Assuming the recommended holding period of the CoCos is one year, the number of trading days in this holding period is $N = 250$. As such the VaR in the calculation becomes a 1-year VaR based on an average 1-year skew and kurtosis. The rescaling factor (T) equals one because no rescaling is necessary in order to express the VEV over a term of 1 year. On the other hand, if the recommended holding period of the CoCo is equal to one trading day ($N = 1$), the skew and kurtosis are not rescaled in Equations 9 and 10. As a result the value for the VaR is a 1-day VaR based on 1-day skew and kurtosis. Here we take the rescaling factor $T = 1/250$. Notice that there is a major difference in the values

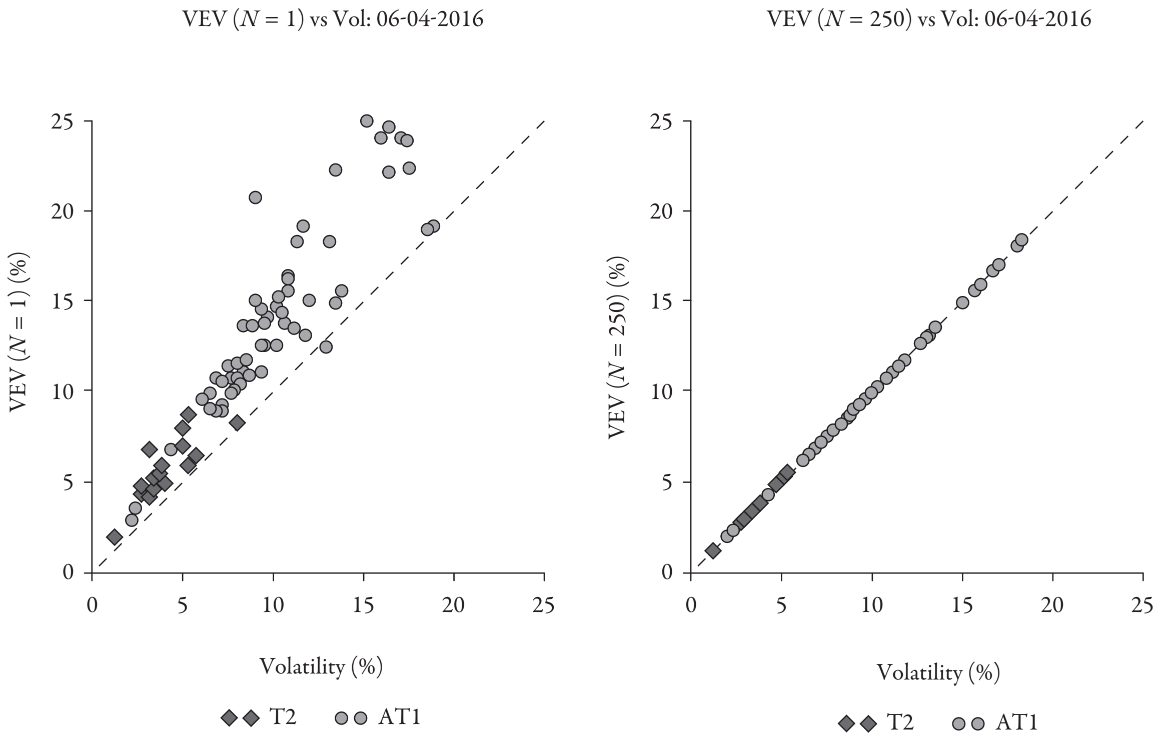


Figure 2: The impact of the recommended holding period on the VEV. A 1-year VEV is displayed with respect to the 1-year volatility (in %) on April 6, 2016 for $N = 1$ (left) and $N = 250$ (right).

of the VEV over a one year term (see Figure 2). The largest difference is observed for the Credit Suisse CoCo (CS 6 09/29/49) which has a VEV of 21.13% and a volatility of 9.11% on April 6, 2016. The impact of the non-normal distribution is best observed for a recommended holding period of one day (or one trading period). This maximises the difference between both measures. Therefore we apply $N = 1$ in the subsequent sections.

3.1.2 *Application of the Cornish-Fisher expansion*

The Cornish-Fisher expansion describes a transformation of a normal distributed variable to a non-normal distributed variable by using the first four moments of the non-normal variable. Suppose Z is a normal distributed variable. The transformation is then given by:

$$X = Z + (Z^2 - 1)\frac{s}{6} + (Z^3 - 3Z)\frac{k}{24} - (2Z^3 - 5Z)\frac{s^2}{36} \tag{14}$$

where X is the a non-normal distributed variable. The parameters s and k can be derived from the actual skewness and excess kurtosis of X. The following relation holds between the central moments of X and the parameters s and k (Maillard, 2012):

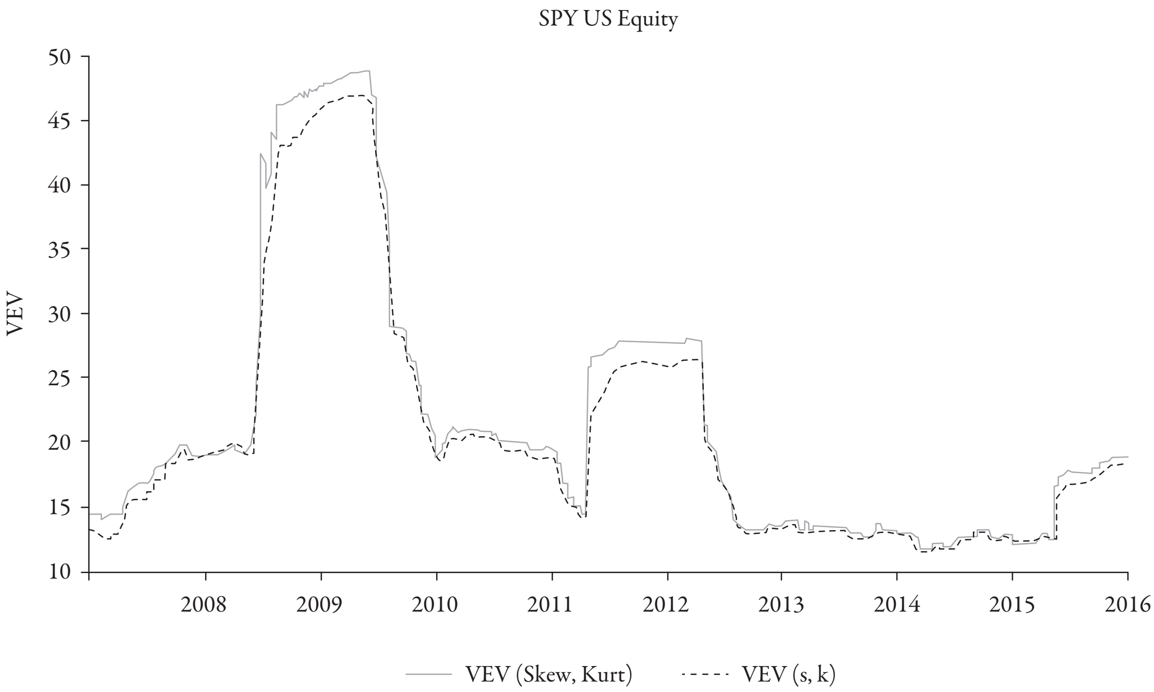


Figure 3: Difference in the VEV for the S&P 500 ETF price based on the skewness and kurtosis parameters and the actual skewness and excess kurtosis.

$$M_1 = 0 \tag{15}$$

$$M_2 = 1 + \frac{1}{96}k^2 + \frac{25}{1296}s^4 - \frac{1}{36}ks^2 \tag{16}$$

$$M_3 = s - \frac{76}{216}s^3 + \frac{85}{1296}s^5 + \frac{1}{4}ks - \frac{13}{144}ks^3 + \frac{1}{32}k^2 s \tag{17}$$

$$\begin{aligned} M_4 = {} & 3 + k + \frac{7}{16}k^2 + \frac{3}{32}k^3 + \frac{31}{3072}k^4 - \frac{7}{216}s^4 - \frac{25}{486}s^6 + \frac{21665}{559872}s^8 - \\ & -\frac{7}{12}ks^2 + \frac{113}{452}ks^4 - \frac{5155}{46656}ks^6 - \frac{7}{24}k^2 s^2 + \frac{2455}{20736}k^2 s^4 - \frac{65}{1152}k^3 s^2 \end{aligned} \tag{18}$$

Notice that the parameter values do not coincide with the effective skewness and kurtosis except for very low values. However the skewness and excess kurtosis are in general not close to zero for financial timeseries since the returns are in general not normal distributed. The impact of the difference is displayed in Figure 3. This is the first common pitfall of the application of the Cornish-Fisher expansion.

The second pitfall is the domain of validity. The formula only holds for a specific range of skewness and kurtosis values. The domain is derived to conserve the order of

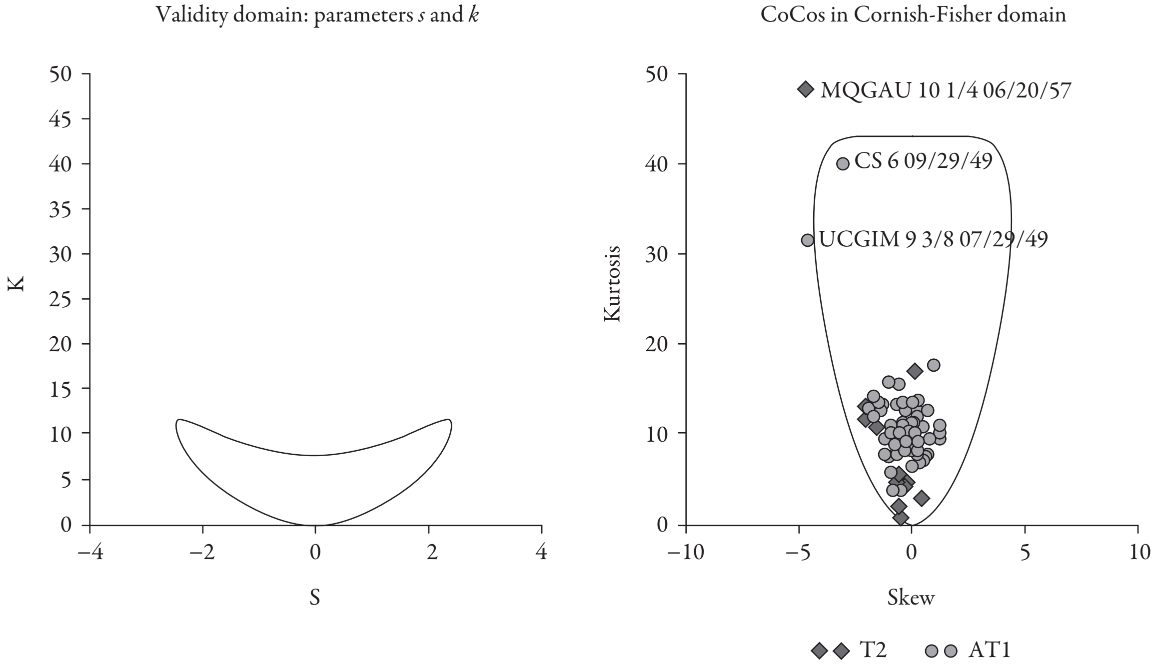


Figure 4: Validity domain of Cornish-Fisher expansion in terms of the parameters (left) and the actual skewness and excess kurtosis (right). A few CoCos are situated outside the validity domain on April 6, 2016.

the quantiles of the distribution. In Maillard (2012), the domain of validity is described in terms of the parameters k and s by:

$$-6(\sqrt{2}-1) \leq s \leq 6(\sqrt{2}-1) \tag{19}$$

$$4+\frac{11}{9}s^2-\frac{4}{6}\sqrt{\frac{1}{36}s^4-6s^2+36} \leq k \leq 4+\frac{11}{9}s^2+ +\frac{4}{6}\sqrt{\frac{1}{36}s^4-6s^2+36} \tag{20}$$

This domain is displayed in Figure 4. The boundaries can be translated in terms of the actual skewness and kurtosis. The CoCos of Table 1 were added to check the applicability of the VEV formula. We see that in general, most CoCos are situated in the validity domain.

3.2 Case study: Risk of different asset classes

In this section, a case study is provided to illustrate differences in risk among the different asset classes. We compare the risk parameters volatility and VEV of multiple Exchange Traded Funds (ETFs) with the Credit Suisse CoCo index. The dataset consists of 20 equity ETFs, 14 mixed ETFs and 21 fixed income ETFs. We observe the values

Table 2: Each PRIIP is classified in a Market Risk Category based on its VEV value

MRI	VEV(%)	Asset Type
1	< 0.5	–
2	0.5-5.0	Fixed Income ETF
3	5.0-12.0	Preferred, CoCo (index) AND Mixed ETF
4	12.0-20.0	Equity ETF
5	20.0-30.0	–
6	30.0-80.0	–
7	> 80.0	–

Table 3: Risk in terms of sigma-events

Sigma	Frequency	Explanation
1σ	1 in 3	Twice a week
2σ	1 in 22	Monthly
3σ	1 in 370	Every year and a half
4σ	1 in 15,787	Twice a lifetime
5σ	1 in 1,744,278	Once a history (5,000 years)

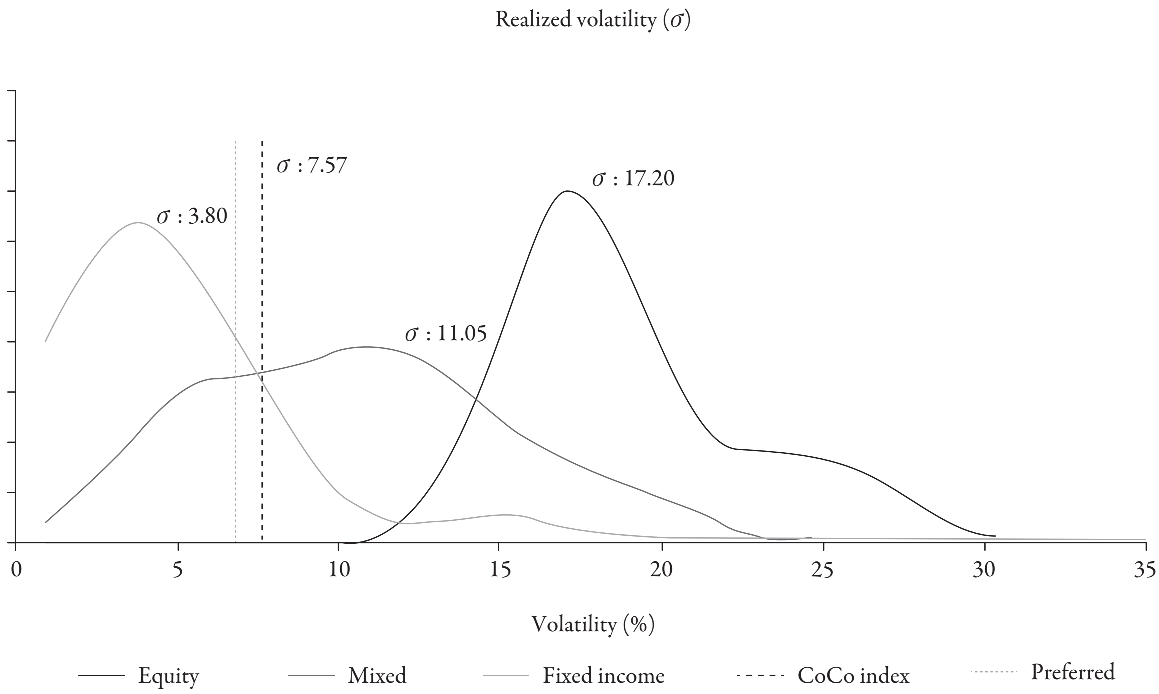
for the VEV on April 6$^{\text{th}}$ 2016 and take an observation window of 250 business days. In Figure 5, a Gaussian kernel density estimator (KDE) gives an overview of how the realised volatilities are distributed. Even with a regulatory measure that takes the fat-tail risk of CoCo bonds into account, the risk profile of CoCos remains distant from the equity risk. The risk of the preferred «Ishares US Preferred Stock» seems to be slightly lower than the CoCo index risk. The question arises that if in times of stress, the VEV of CoCos can enter into the range of the equity class.

In the PRIIPS regulation (European Commission, 2017) each asset (PRIIP) is assigned to a specific market risk measure (MRM) class according to its VEV value. From the distributions of the previous exercise, we can observe which MRM class is best fitted for each asset type (see Table 2). The MRM class should be increased by one additional level if the PRIIP has only monthly price data. The CoCo index belongs to the market risk category 3 together with the Preferred ETF and the Mixed ETFs. This does not mean their is no difference in their risk profiles. For Deutsche Bank and the Banco Popular CoCo, the VEV moves up to risk category 6 (see Figure 6).

4 Are CoCos Moving Out of Sync?

Market analysts often think about risk in terms of sigma-events. These events can be translated into a frequency of occurrence as shown in Table 3. However in 2008 markets experienced 25-standard deviation events that occurred several days in a row. Also in terms of the daily returns of the first quarter of 2016, the CoCo asset class observed extreme conditions compared to the returns of 2015.

The sigma-events are typically related to z-scores (standardised values) which are often used for univariate outlier detection for continuous variables. In Figure 7, we show the



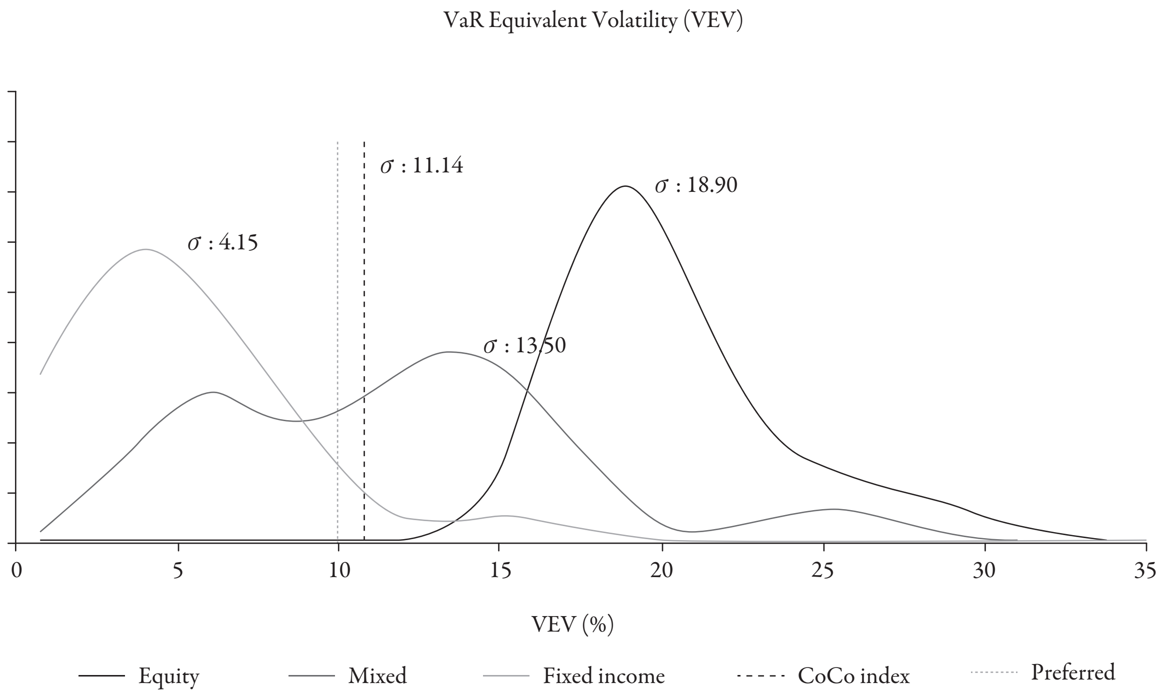
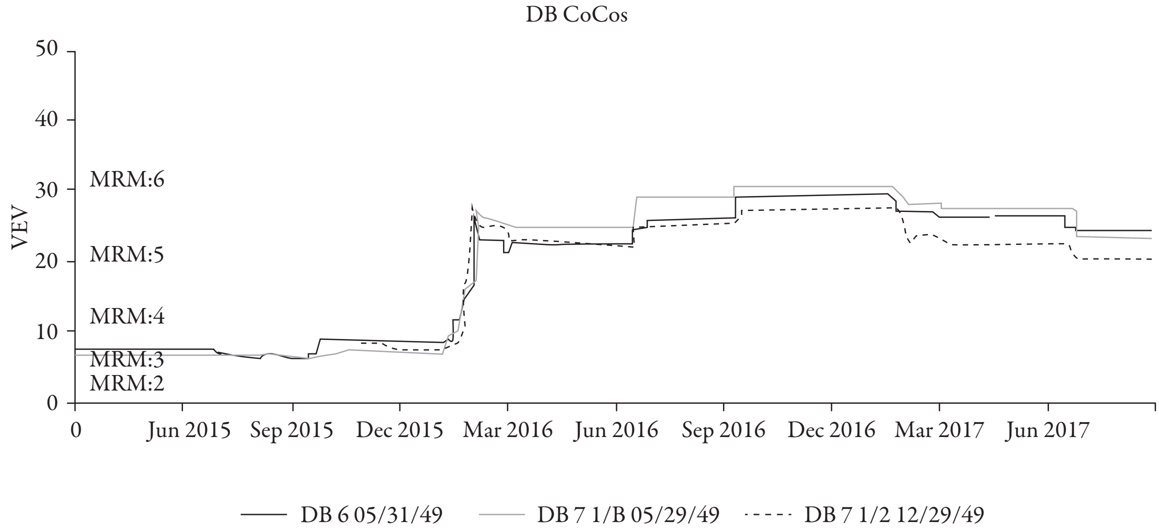


Figure 5: Gaussian KDE of the 1-year volatility (above) and the 1-year VEV (below) per asset class compared with the Credit Suisse CoCo Index on April 6, 2016.

average CoCo volatility and VEV over time for the AT1 CoCos and T2 CoCos. A clear increase in both risk measures can be seen during the first quarter of 2016. The asymmetric tail risk causes a higher increase in the VEV risk measure compared to the volatility.



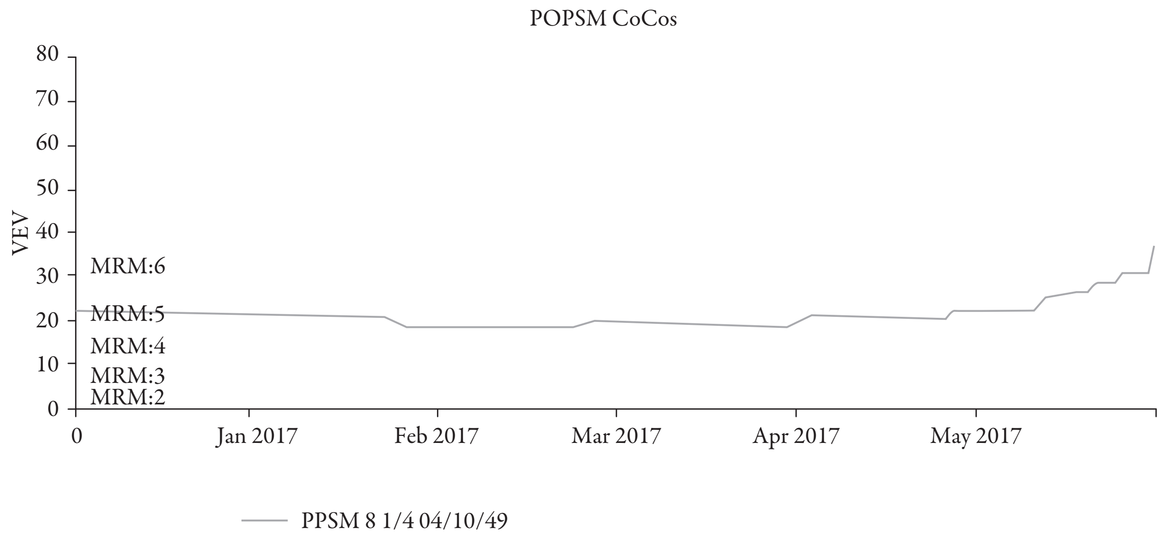


Figure 6: Moves of the VEV of the Deutsche Bank and Banco Popular CoCos over multiple market risk categories.

The real issue is however if the CoCo bonds move into the area outside the risk defined in the contract such as relating to their sensitivity with the underlying share price return. Did something clearly broke down at the start of 2016? Or were CoCos following the price performance of the bank shares? To see whether Q1 2016 was an outlier, we should not only look at the CoCo bond returns but also take into account the share price returns. In Figure 8, we show a scatterplot of the daily returns of the Credit Suisse CoCo index versus the Stoxx Banking Index for the different years. Instead of using a volatility measure in one dimension (σ) we will use a covariance matrix Σ of the equity returns and CoCo market returns. In the following sections, we explain our approach in a higher dimensional space and apply it to detect outliers in the CoCo market.

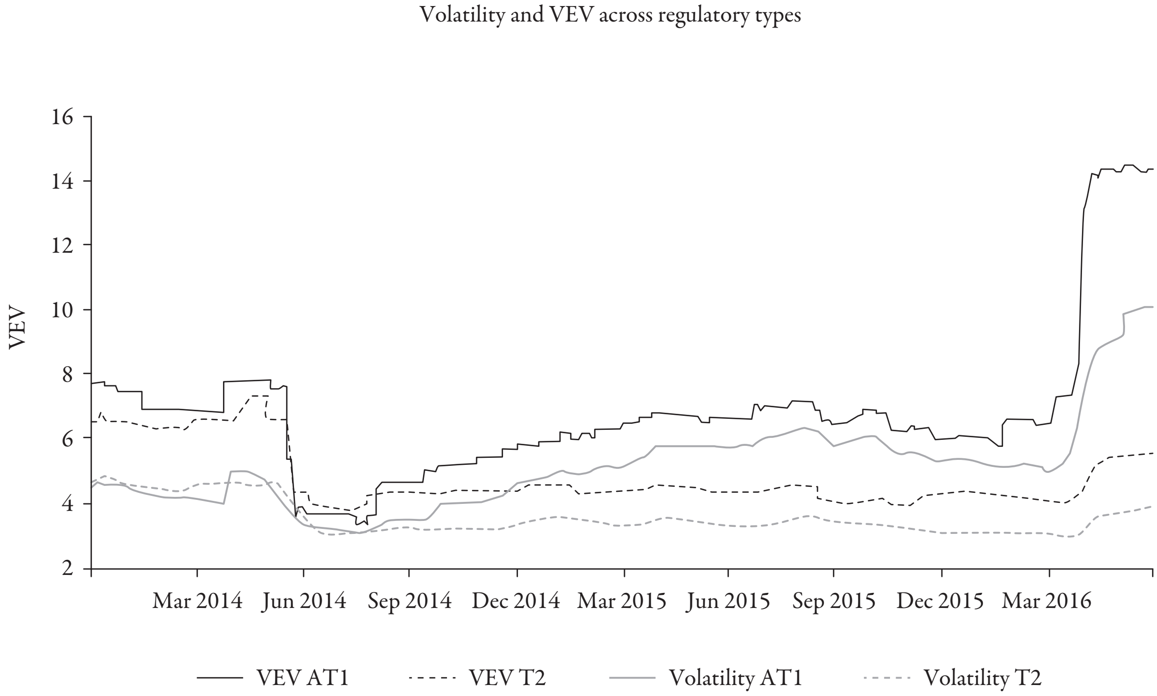


Figure 7: Average volatility and VEV for AT1 CoCos and T2 CoCos.

CoCo bonds can be seen as derivative instruments with some capital ratio (CET1) as underlying driver. This ratio is put forward by the regulator as a measure for the health of a financial institution, for example in the ECB stress tests. Hence the CoCo market price, as the price of a derivative, contains forward looking information or at least the market's anticipated view on the financial health of the institution. In this context we can interpret the outlying behaviour of a CoCo price as a warning for financial (un-) healthiness.

4.1 Multivariate Covariance Determinant (MCD)

Outliers in a multivariate setting are no longer defined as a z-score but as a Mahalanobis Distance (MD). This distance measures a point x versus a data cloud X as defined by:

$$MD_X(x) = \sqrt{(x-\mu_x)\Sigma^{-1}(x-\mu_x)^T} \tag{21}$$

where Σ denotes the covariance matrix of X. Intuitively, the x–μ_X shows how far a point x stands away from the center of the cloud. In the meantime Σ explains the spread on the dataset X. The distance is hence based on the correlation between the variables. It measures the connectedness of two sets with multiple variables. The distance reduces to the Euclidean distance if the covariance matrix is the identity matrix, and the normalised Euclidean distance if the covariance matrix is diagonal. The MD measures how many sigma-events a data point is away from the center of a multivariate distribution (Hoyle *et al.* 2016).

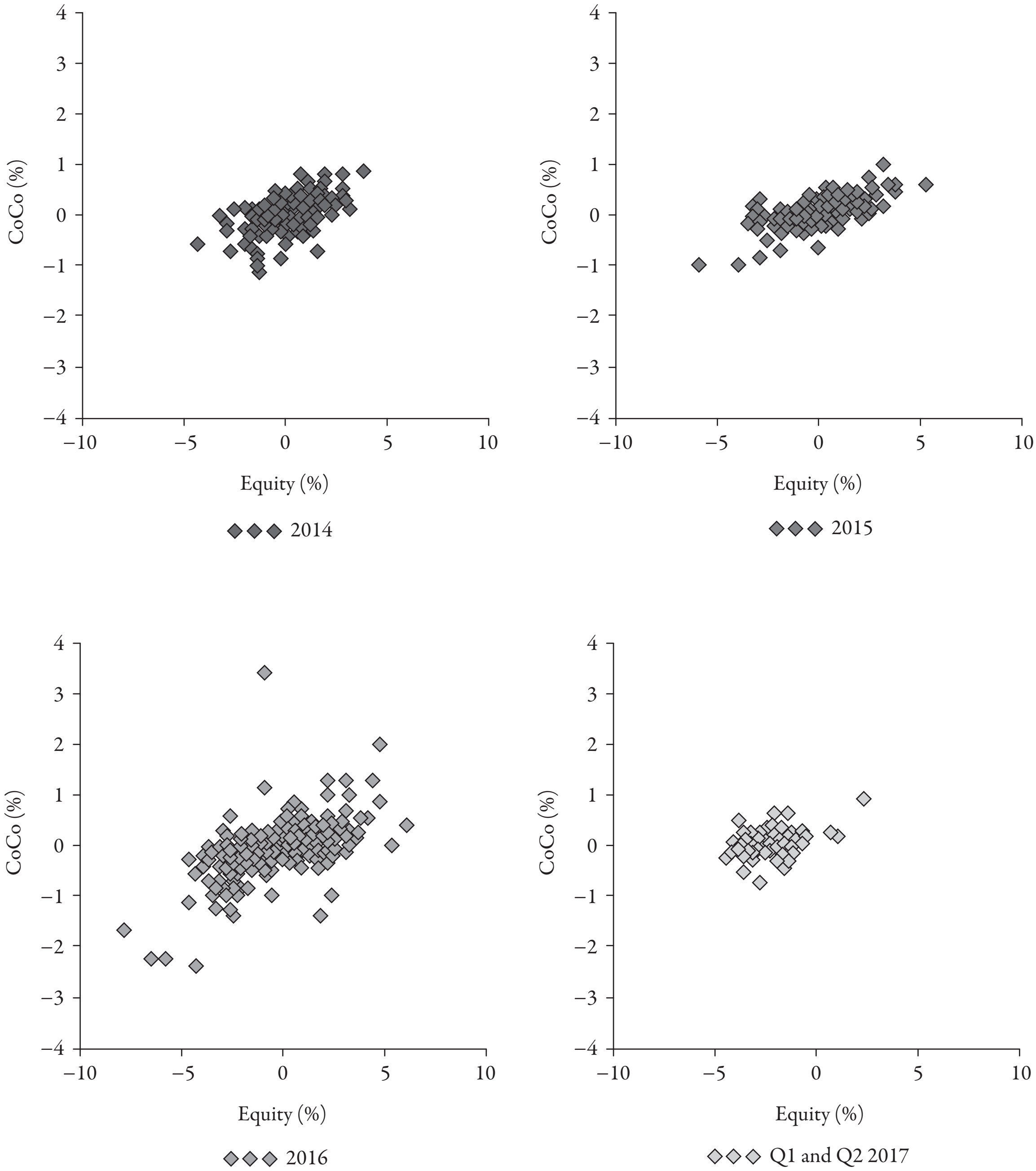


Figure 8: Scatterplot of the daily returns in the Credit Suisse CoCo index versus daily returns in the Stoxx Banking Index.

The Mahalanobis distance can be used to find outliers in multivariate data. The squared Mahalanobis distance is chi-squared distributed with p degrees of freedom under the assumption that the p-dimensional dataset is multivariate normal distributed. For a sample of size n, we denote each observation by $x_i \in R^p$ with $i = 1; \ldots; n$. The estimated Mahalanobis distance is denoted with $MD_X(x_i)$. Afterwards the squared MD is compared with the quantiles of the chi-squared distribution with p degrees of freedom. For example, if the squared MD is larger than the 99% quantile, the observation can be classified as a potential outlier.

Notice that this distance measure is very sensitive to outliers itself. Single extreme observations, or groups of observations, departing from the main data structure can have a severe inuence to this distance measure. Both the location and covariance are usually estimated in a non-robust manner. In order to provide reliable measures for the recognition of outliers, one should apply a more robust measure for location and covariance. In practice classical fitting methods used to detect outliers are often so strongly affected by outliers that the resulting fitted model is not able to detect deviating observations. This phenomenon is called the masking effect (Rousseeuw *et al.*, 2006).

Different solutions exist to make the distance measure less inuenced by outliers or more robust. One approach is to apply the Minimum Covariance Determinant (MCD) method. MCD is a commonly-used robust estimate of dispersion which can be used to construct robust MDs. The MCD estimator is a computationally fast algorithm introduced in Rousseeuw and van Driessen (1999).

Using robust estimators of location and scatter in formula (22) leads to so called robust distances. In Rousseeuw and van Zomeren (1990), the robust MD is used to derive a measure of outlier size. The first part in the derivation of the robust Mahalanobis distance is the concentration step. The dataset is divided into different non-overlapping subsamples. For each subsample the mean and the covariance matrix is computed in each feature dimension of the subsample (see Hubert and Debruyne, 2009). The MD is computed for every multidimensional data vector x_i. Afterwards, the data are ordered ascendingly by this distance in each subsample. Next, subsamples with the smallest MD are selected from the original samples. This procedure is iterated until the determinate of the covariance matrix converges (see Hoyle *et al.*, 2016). Hence the robust measure first selects a subset of the original data whose classical covariance has the lowest determinant. The determinant of a covariance matrix indicates how much space the data-cloud takes. The second part is a correction step to compensate for the fact that the estimates were learned from only a portion of the initial data (Pison *et al.*, 2002). This robust Mahalanobis distance also assumes the data is multivariate normally distributed and does not account for the sample size of the data.

Hardin and Rocke (2005) showed that the cut-off value derived from the chi-squared distribution (i.e. $\chi^2_{p,(1-\alpha)}$) is based on the asymptotic distribution of the robust distances. This often indicates too many observations are deemed to be outliers which means that test results show more false-positive detections than expected for robust MD. In Hardin and Rocke (2005), the corrected distribution of the robust distances is approximated by the following F-distribution:

$$\frac{np}{1-p} MD_X^2(x) \sim F_{p,n-p} \tag{22}$$

Cerioli (2010) also rejects the chi-square quantiles for the detection of outliers. The author developed a new calibration methodology, called Iterated Reweighted MCD (IRMCD), which provides outlier detection tests with the correct Type I error behaviour for the robust MD. In Green and Martin (2014) an extension is developed that combines the method of Hardin and Rocke with the IRMCD method of Cerioli.

4.2 Measuring the outliers

An application of this method is the study of irregular behaviour in the relationship between equity returns versus CoCo bond returns. The detection of irregular behaviour will guide us to possible dislocations and potential stability risks.

4.2.1 *Outliers compared to previous year*

We compare the daily returns in year T with the previous year $T - 1$. In Figure 9, we display the robust Mahalanobis distance over the year 2016 (resp. 2017) where we train the data on the returns of stock price and the CoCo price index in the year before: 2015 (resp. 2016). The data points with an extreme distance compared to the overall group are marked and mentioned in the legend.

In February 2016 a general fear over Europe's banking industry was observed and concerns were raised about Deutsche Bank's ability to pay off the high coupon values of CoCos. During certain days the CoCos moved extremely compared to the historical CoCo price returns and their underlying equity returns. In June 2016 the outlier detection method highlights the Brexit election as an outlying phenomenon.

Also, the outliers of 2017 can be related to market circumstances. The first outlier in March 2017 was caused by UniCredit due to uncertainty in its next AT1 coupon payment. On April 24th 2017 the outcome of the French election lighted up the EU Bank Stoxx indicating a second outlier. The latest outlier in June 2017 is probably related to the UBS shares drop down due to concerns over margins in its wealth management division. This impacted the Stoxx Banking index whereas overall CoCo prices remained stable. The general CoCo market has proven resilient while losses were imposed on Banco Popular bondholders in June 2017.

4.2.2 *Outlier detection per issuer*

In the next step we detect outlying behaviour of CoCos from specific issuers. The model is fit to the CoCo price return and the underlying equity return during a 90-day history window. In Figure 10, the averaged daily robust MD is shown for different issuers. During certain days these observations move extremely compared to the previous 90-day time period.

It is worthy to note that Deutsche Bank had to reassure its coupon payments of its outstanding CoCos during the first quarter of 2016. Cancellation of the high coupons in a CoCo would be a significant loss for the CoCo investor. This is also observed in the robust distance of Deutsche Bank that moves out of the barrier derived from the 99% quantile of the F-distribution for a longer time period. In the beginning of 2016, other CoCo distances moved also out of this boundary. In February 2016 the CoCo market experienced large losses. During certain days the CoCos moved extremely compared to the historical CoCo price returns and their underlying equity returns.

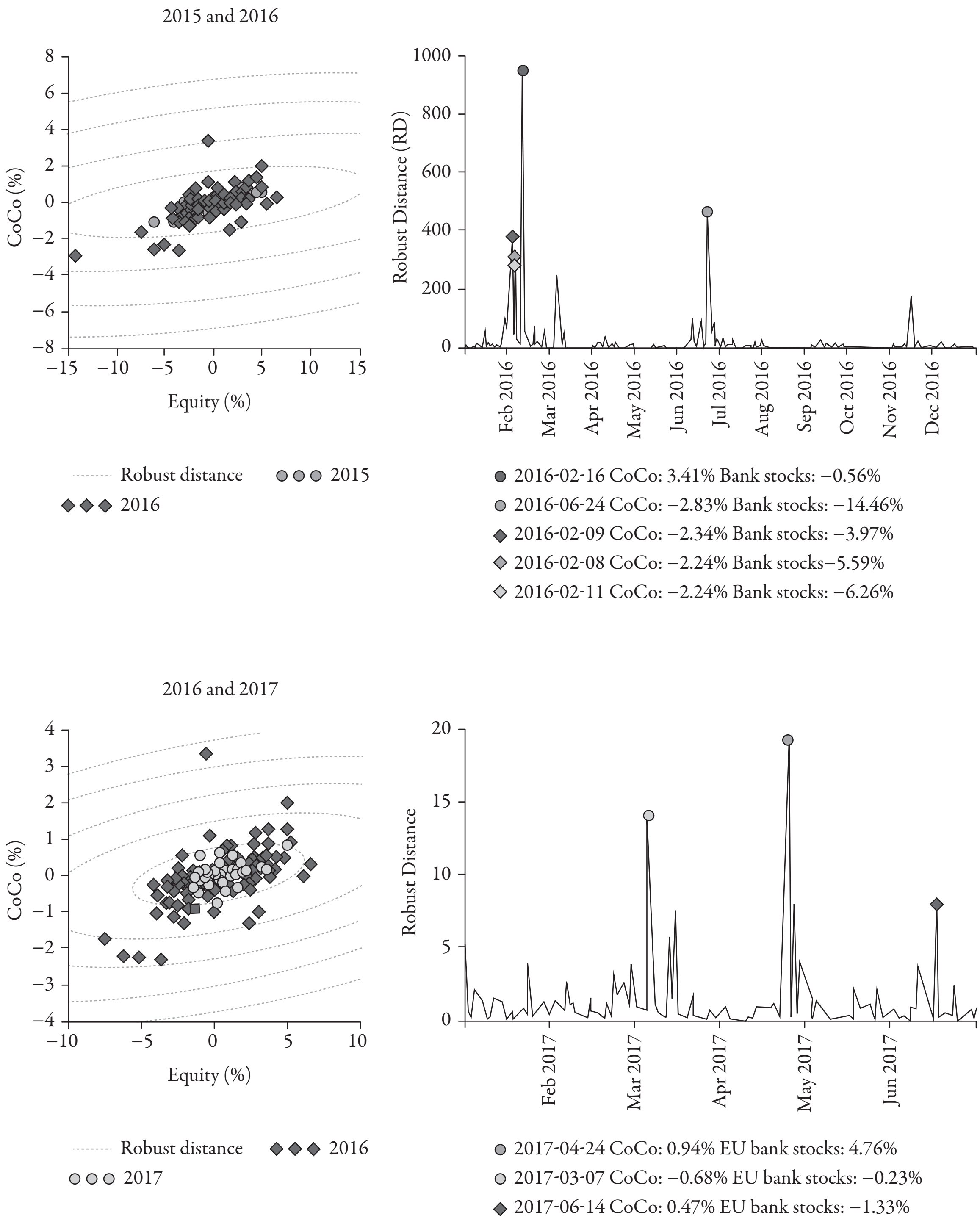


Figure 9: Left: Scatterplot of the daily returns in the CoCo Credit Suisse Index versus daily returns in the Stoxx Banking Index, right: Robust Mahalanobis Distance.

On June 6th 2017 the European Central Bank considered the bank Banco Popular to be «failing or likely to fail» (European Central Bank, 2017). This classification is used by supervisors to indicate institutions that become non viable. The Single Resolution Board stepped in to force the sale to Santander. As part of the deal Banco Popular's junior bonds were wiped out including its CoCo bonds. That marks the first write-off of CoCos industrywide since regulators developed the bonds in the wake of the financial

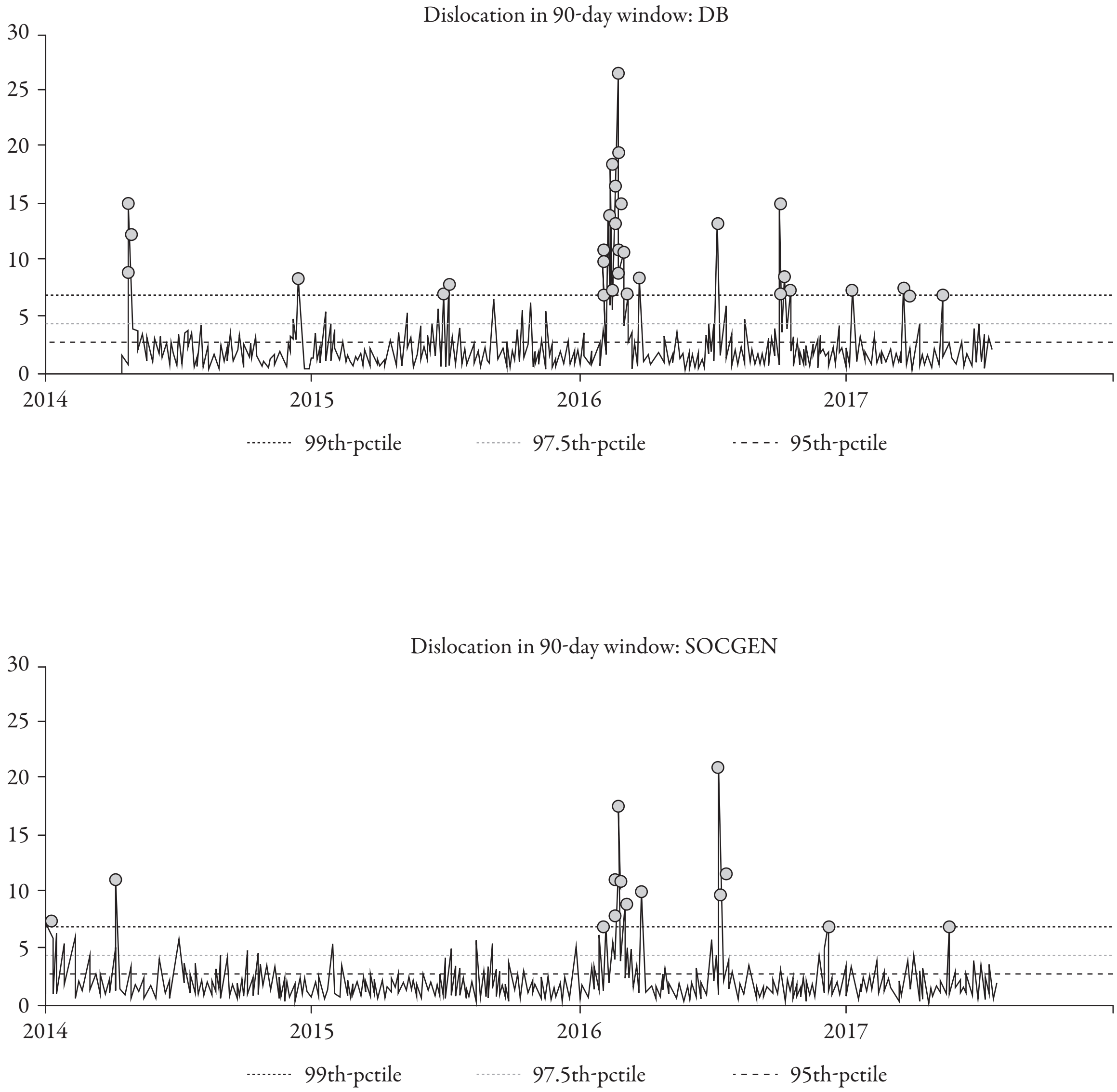
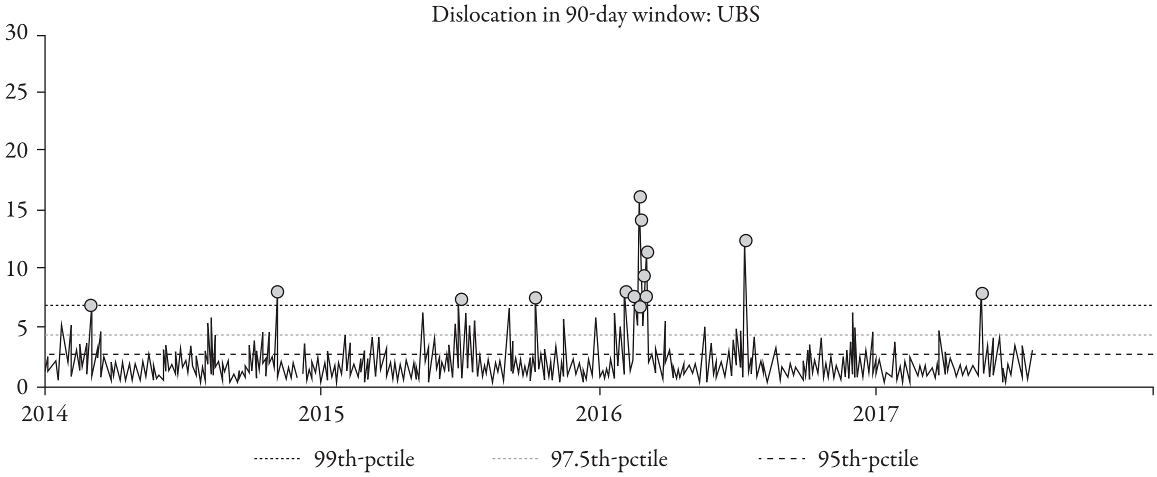


Figure 10a: Robust Mahalonbis Distance for CoCos averaged per issuer.

crisis. The sale spared Spain's taxpayers the cost of another bailout. Banco Popular's CET1 fully loaded ratio, stood at 7.33 percent in March, one of the weakest among European lenders. The remaining market for AT1 bonds remained stable after the take-over.

This corresponds with the outlying behaviour of the Banco Popular (POPSM) CoCo. The robust distance detects periods of stress for the bank starting in mid 2015. Also during the first quarter of 2016, Banco Popular remains outside its boundaries for a longer time period compared to other banks. On April 3rd 2017 the robust distance of Banco Popular rose again above the indicated boundary. From that point onwards, the distance moved above the boundary in all the upcoming weeks multiple times. Hence the bailout of Banco Popular in June 2017 was not unexpected from this data mining exercise.



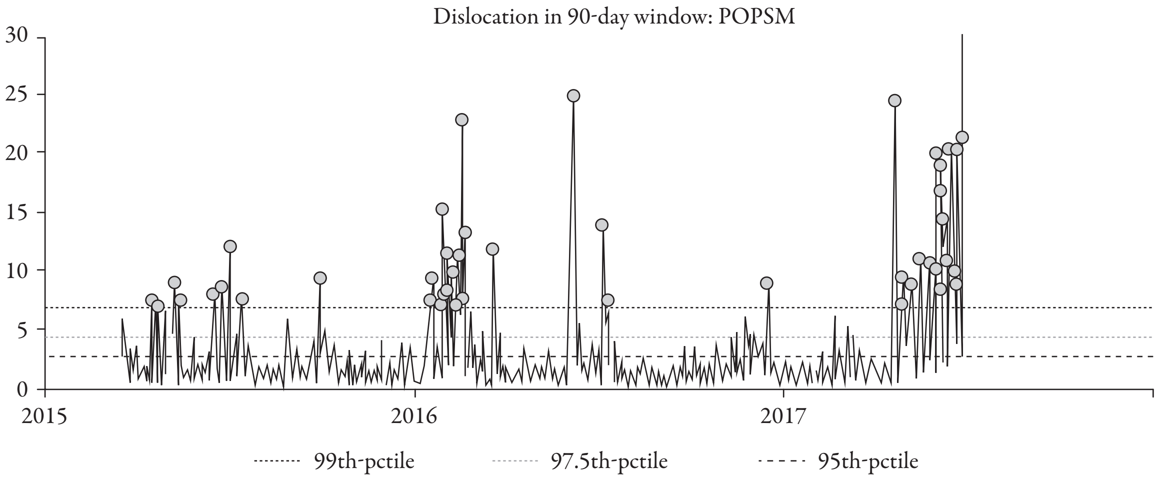


Figure 10b: Robust Mahalonbis Distance for CoCos averaged per issuer.

5 Conclusion

The new risk measure, called VEV, entered the PRIIPs guidelines and takes into account fat-tail and skewed distributions. By application of this measure as described by the regulation, one should be aware of the impact regarding how the length of the recommended holding period is defined. When the period length is set too high, the VEV will be equivalent to volatility. The VEV measure denotes however that the CoCos behaved as described by their hybrid nature between the equity and fixed income asset types.

On the financial markets, we observe extreme CoCo price moves together with extreme moves in the underlying equity. This relation is clear by construction of the CoCo asset class. However, the detection of outliers in the CoCo market should be performed by taking into account multiple variables like the CoCo market returns and the underlying equity return. The main contribution of this paper lies in the fact that we study the movement of the price of a CoCo bond in two dimensions.

The outliers in the CoCo market are caught across these two dimensions using a robust Mahalanobis distance measure. Based on this robust multiple-dimension distance, we can detect CoCos that are outlying compared to previous time periods but also by taking into account extreme moves of the market situation. We detected the Deutsche Bank CoCo and the Banco Populare CoCo to be outliers using the MCD algorithm. Indeed Deutsche Bank had to reassure its coupon payments of its outstanding CoCos during the first quarter of 2016. On June 7th 2017 the first write-off of CoCos industry-wide has occurred for Banco Popular CoCo since the initiation of these instruments. Every investor in CoCos should also be aware that the high coupon is a compensation for the high risks involved.

References

Allen L. and Tang Y. (2016) 'What's the Contingency? A Proposal for Bank Contingent Capital Triggered by Systemic Risk', *Journal of Financial Stability*, 26, pp. 1-14.

Avdjiev S., Bogdanova B. and Kartasheva A. (2013) 'Bis Quarterly Review', in C. Borio, D. Domanski, C. Upper, S. Cecchetti and P. Turner (eds), *International Banking and Financial Market Developments, Chapter Cocos: A Primer*.

Basel Committee on Banking Supervision (2010) 'Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems', Technical report, Bank for International Settlements.

Basel Committee on Banking Supervision (2011) 'Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems', Technical report, Bank for International Settlement.

Brinner M. (1974) 'On the Stability of the Distribution of the Market Component in Stock Price Changes', *Journal of Financial and Quantitative Analysis*, 9, pp. 945-961.

Campolongo F., De Spiegeleer J., Di Girolamo F. and Schoutens W. (2017) 'Contingent Conversion Convertible Bond: New Avenue to Raise Bank Capital', *International Journal of Financial Engineering*, 4.

Cerioli A. (2010) 'Multivariate Outlier Detection with High-Breakdown Estimators', *Journal of the American Statistical Association*, 105 (489), pp. 147-156.

Corcuera J., De Spiegeleer J., Fajardo J., Jönsson H., W., S. and Valdivia A. (2014) 'Close Form Pricing Formulas for CoCa CoCos', *Journal of Banking and Finance*.

Cornish E.A. and Fisher R.A. (1938) 'Moments and Cumulants in the Specification of Distributions', *Review of the International Statistical Institute*, 5 (4), pp. 307-320.

Culp C. (2009) 'Contingent Capital vs. Contingent Reverse Convertibles for Banks and Insurance Compagnie', *Journal of Applied Corporate Finance*, 21 (4), pp. 17-27.

De Spiegeleer J., Höcht S., Marquet I. and Schoutens W. (2017) 'CoCo Bonds and Implied CET1 Volatility', *Quantitative Finance*, 17 (6), pp. 813-824.

De Spiegeleer J. and Schoutens W. (2012) 'Steering a Bank around a Death Spiral: Multiple Trigger CoCos', *Wilmott Magazine*, 2012 (59), pp. 62-69.

De Spiegeleer J. and Schoutens W. (2013) 'Multiple Trigger CoCos: Contingent Debt without Death Spiral Risk', *Financial Markets, Institutions & Instruments*, 22 (2).

De Spiegeleer J. and Schoutens W. (2014) 'CoCo Bonds with Extension Risk', *Wilmott Magazine*, 71, pp. 78-91.

De Spiegeleer J., Schoutens W. and Van Hulle C. (2014) *The Handbook of Hybrid Securities: Convertible Bonds, CoCo Bonds and Bail-in*, Willey, chapter 3.

European Central Bank (2017) 'ECB Determined Banco Popular Espanol S.A. Was Failing or Likely to Fail', PRESS RELEASE.

European Commission (2017) 'Annexes to the commission Delegated Regulation (EU)', Technical report, *Offcial Journal of the European Union*.

Flannery M.J. (2009) 'Stabilizing Large Financial Institutions with Contingent Capital Certificates', Technical report, Graduate School of Business Administration, University of Florida.

Green C.G. and Martin R.D. (2014) 'An Extension of a Method of Hardin and Rocke, with an Application to Multivariate Outlier Detection Via the IRMCD Method of Cerioli', Working Paper.

Hardin J. and Rocke D.M. (2005) 'The Distribution of Robust Distances', *Journal of Computational and Graphical Statistics*, 14 (4), pp. 928-946.

Hoyle B., Raul M.M., Paech K., Bonnett C., Seitzl S. and Weller J. (2016) 'Anomaly Detection for Machine Learning Redshifts Applied to SDSS Galaxies', *Monthly Notices of the Royal Astronomical Society*.

Hubert M. and Debruyne M. (2009) 'Minimum Covariance Determinant', *Wiley Interdisciplinary Reviews: Computational Statistics*, 2 (1), pp. 36-43.

Jorion P. (2006) *Value at Risk: The New Benchmark for Managing Financial Risk*, 3 edn, McGraw-Hill Education-Europe.

J.P. Morgan/Reuters (1996) 'RiskMetrics TM – Technical Document', Technical report, Morgan Guaranty Trust Company & Reuters Ltd. Fourth Edition.

Liaw A. and Wiener M. (2002) 'Classification and Regression by Randomforest', *R News*, 2 (3), pp. 18-22.

Liu F., Ting K. and Zhou Z.-H. (2012) 'Isolation-Based Anomaly Detection', *ACM Transactions on Knowledge Discovery from Data*, 6 (1).

Maes S. and Schoutens W. (2012) 'Contingent Capital: An In-Depth Discussion', *Economic Notes*, 41 (1/22), pp. 59-79.

Maillard D. (2012) 'A User's guide to the Cornish Fisher Expansion', Technical report, Conservatoire National des Arts et M_etiers (CNAM); Amundi Asset Management. Available at SSRN: https://ssrn.com/abstract=1997178.

Mandelbrot B. (1963) 'The Variation of Certain Speculative Prices', *The Journal of Business*, 36 (4), pp. 394-419.

McDonald R.L. (2013) 'Contingent Capital with a Dual Price Trigger', *Journal of Financial Stability*, 9 (2), 230-241.

Pison G., Van Aelst S. and Willems G. (2002) 'Small Sample Corrections for LTS and MCD', *Metrika*, 55, pp. 111-123.

Rousseeuw, P.J., Debruyne M., S., E. and M., H. (2006) 'Robustness and Outlier Detection in Chemometrics', *Critical Reviews in Analytical Chemistry*, 36, pp. 221-242.

Rousseeuw P. J. and van Driessen K. (1999) 'A Fast Algorithm for the Minimum Covariance Determinant Estimator', *Technometrics*, 41 (3), pp. 212-223.

Rousseeuw P.J. and van Zomeren B.C. (1990) 'Unmasking Multivariate Outliers and Leverage Points', *Journal of American Statistics Association*, 58, pp. 633-651.