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Abstract

Financial inter-linkages play an important role in the emergence of financial instabilities and the formulation of systemic risk can greatly benefit from a network approach. In this paper, we focus on the role of linkages along the two dimensions of contagion and liquidity, and we discuss some insights that have recently emerged from network models. With respect to the issue of the determination of the optimal architecture of the financial system, models suggest that regulators have to look at the interplay of network topology, capital requirements, and market liquidity. With respect to the issue of the determination of systemically important financial institutions the findings indicate that both from the point of view of contagion and from the point of view of liquidity provision, there is more to systemic importance than just size. In particular for contagion, the position of institutions in the network matters and their impact can be computed through stress tests even when there are no defaults in the system.

Keywords: Systemic Risk, Financial Networks, Financial Contagion, Financial Distress, Default Cascade, Systemically Important Financial Institutions, Debtrank, Controllability.
JEL Codes: G01; G21; G33; G28; E58.

1 Introduction

Systemic risk in finance denotes in general the risk of collapse of a major part of the financial market with the disruption of critical functionalities. In this paper, we mean more specifically the risk of default or distress of a large part of the financial institutions. There is growing consensus around the idea that financial inter-linkages play an important role in the emergence of financial instabilities, and that the mathematical formulation of systemic risk can greatly benefit from a network approach (Haldane, 2009).

The first reason why linkages matter is that they can have ambiguous effects: on the one hand, they increase individual profitability and reduce individual risk, but on the other hand, they propagate contagion and distress, thus increasing systemic risk. On this topic several issues remain open but much work has already been done in recent years (Allen *et al.*, 2011; Allen and Gale, 2000; Battiston *et al.*, 2012a,b; Beale *et al.*, 2011; Cont *et al.*, 2011; Gai *et al.*, 2011; Gai and Kapadia, 2010; Greenwald, Bruce and Stiglitz,

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1993; Haldane and May, 2011; Stiglitz, 2010). In addition, to be relevant for contagion interlinkages are also relevant for liquidity provision (Gai *et al.*, 2011). The second reason is more subtle and seldom formalized. On the one hand, institutions have an incentive to become too-connected-to-fail and too-correlated-to-fail (Acharya, 2009). They thus form tightly knit structures (Boss *et al.*, 2004; Cajueiro and Tabak, 2008; Craig and Von Peter, 2010; De Masi *et al.*, 2006; Iori, De Masi, Precup, Gabbi and Caldarelli *et al.*, 2008; Soramäki *et al.*, 2007; Upper and Worms, 2004; Vitali *et al.*, 2011) and gain exposures to similar risks (Gai and Kapadia, 2010). On the other hand, once these structures emerge in the financial system, they alter the incentives of each institution with respect to risk taking, and they may provide groups of institutions with market power or put them in the position to influence the debate on regulation.

In this paper, we leave out the strategic valence of financial linkages and we focus on their role along the two dimensions of contagion and liquidity. Our aim is to discuss some issues and insights from network models that have recently emerged regarding in particular: 1) the optimal architecture of financials system and 2) the determination of the systemically important financial institutions.

The paper is organized as follows. In the rest of the Introduction we provide an overview of the network approach to modeling financial systems, some relevant literature from other fields that helps us understanding the problem in a broader context, and a summary of the results. In Section 2 we examine the role of liquidity in default cascades and the insights we learn about architectures. In Section 3 we discuss a recent method to assess the systemic impact of financial institutions in terms of distress. In Section 4 we discuss a complementary method to assess the systemic impact of financial institutions in terms of liquidity. In Section 5 we draw conclusions.

1.1 Interacting Networks in the Financial System

The financial system can be regarded as a network in the following way. Financial institutions can be represented as nodes and financial dependencies due to contracts between counterparties or balance-sheet interlocks can be represented as links. Whenever the identity of the counterparties, their past relations and their financial fragility matter, thinking of the system as a network improves our understanding (e.g. in comparison to thinking of it as a simple market place where prices incorporate all the relevant information). Moreover, it is crucial to add securities to this picture, represented as a second type of node. The fact that institutions invest in a given security or are exposed to its price variation via some (publicly traded) derivative contract can also be represented as a second type of link. This general network representation covers for instance the case of a system consisting of an interbank market (where banks are connected via balance-sheet interlocks because the assets of one are the liabilities of some other ones) and an external assets market (where banks are connected through financial instruments issued outside the financial network). In particular, examples of classes of external assets are: (1) mortgage-backed securities played a role in the subprime crisis in 2008-2009; (2) sovereign bonds are playing a role in the current period 2011-2012; (3) links arising through underlying real assets, e.g., the price of housing. It is very important to notice that distress propagates not only via institution-institution linkages but also via institution-security linkages. For instance, the fire-selling of one institution has negative externalities on those institutions that are exposed to the same asset or asset class. This institution may then propagate distress both to the institutions that are exposed to them and to the securities they are exposed to. Many of the effects that are relevant for systemic instabilities can be described within the above general network representation.

Overall then, in this setting, there are two types of connections among banks, both conducive of the spreading of financial distress. One the one hand, shocks move from a bank to another via the direct interlocks between balance sheets. That is, since the liabilities of one bank are the assets of some other banks, the default of the debtor may be better implies a loss for the creditors, as we will see in Section 2. However, as we will see later on in Section 3, even if the obligor does not default, some distress spreads to the creditors anyway. Indeed, the fact that the equity of the obligor is being depleted implies that the market value of its obligations decreases. Notice that complementary to contagion, but not less important, is the issue of liquidity provision. Indeed, in case creditors decide to hoard liquidity rather than providing it to other market players, this has negative externalities to the other institutions and to the system hampering its functionality. Accordingly, the identification of the institutions that are systemically important has to account not only for the potential contagion an institution may cause but also for their role as liquidity providers, as we discuss in Section 4.

On the other hand, there are indirect connections among banks due to the fact that they invest in common assets. This implies that, for instance, if as a result of a shock on the price of an asset, a bank sells a quantity of that asset sufficient to move down the price, the other banks holding the same asset will experience both the initial shock and the secondary shock and may start in turn to sell the asset themselves, triggering a devaluation spiral.

It has been argued that many financial crises originate from bubbles in some asset market, typically assets associated with the housing sector (Alessi and Detken, 2011). Accordingly, in the banking crisis of 2008, the overlapping portfolios channel played a major role in triggering the downturn. Therefore, the ultimate objective of contagion models should be to incorporate both the effects coming from overlapping portfolios and balance sheet interlocks. However, more attention should be devoted to the fact that the interlocking balance sheet channel can greatly amplify the effects of shocks acting along the overlapping portfolio channel. Moreover, from a mathematical point of view, the network of banks and assets is a bipartite network meaning that there are no edges between the nodes in the same class – indeed assets do not invest in other assets. This network can be «projected» into a network of banks in which the relations now represent the fact that two banks hold one or more assets in common, the weight of the edge reflecting the magnitude of the overlap. As it will be argued in more detail later on, although the economic mechanism behind is completely different, there are formal analogies in the spreading of contagion in the two types of networks and lessons to be learnt on one can provide insight for the other. In light of these considerations, in the following section we will focus on the contagion along the balance sheet interlock and we will give insights on how the overlapping portfolios issue can be incorporated.

1.2 Failure Cascades in Complex Systems

The dynamics of default cascades on financial networks as we described earlier is formally equivalent, or very similar, to a number of dynamical processes that have been long studied in other fields. In cascading dynamics, some network nodes are assumed to fail at the beginning of the process. Their failures increase the load (or the level of distress) of the neighboring nodes. When this load at a node exceeds its threshold (i.e. its individual robustness) the node fails, possibly triggering a cascade. This type of propagation dynamics has been initially studied in paradigmatic models such as the sandpile model (Bak *et al.*, 1988) and the Bak Sneppen model (Bak and Sneppen, 1993) and later on in more specific models applied to a series of different situations, ranging from earthquakes to fractures, or species extinctions.

Interestingly there are also some early applications of very similar models to social contexts as for instance in models of social activation (Granovetter, 1978; Watts, 2002). In particular, the problem of characterizing the cascade size (i.e. the number of nodes eventually failing in this process) and its probability distribution is the natural counterpart of the notion of systemic risk in the financial system. Regarding cascade size several analytical investigations have been carried out (Gleeson and Cahalane, 2007), including the effect of heterogeneity in the thresholds (Lorenz *et al.*, 2009) and the cases of degree-correlated networks (Payne *et al.*, 2009), clustered networks (Hackett *et al.*, 2011), and multiplex networks (Brummitt *et al.*, 2012). A large body of work has investigated numerous variants, including: (*a*) the propagation of fractures in a system of fibers (Kim *et al.*, 2005); (*b*) the case in which the load at every node is the total number of shortest paths passing through the node (Crucitti *et al.*, 2004; Motter and Lai, 2002); (*c*) the case in which links (rather than nodes) topple (Moreno *et al.*, 2007); (*d*) cascades of rewiring of links leading to self-organized scale-free networks (Bianconi and Marsili, 2004); (*e*) the sandpile model (Goh *et al.*, 2003), as well as its variant on several interdependent networks (Brummitt *et al.*, 2012); (*f*) the percolation process in interdependent networks (Buldyrev *et al.*, 2010). Most of the attention in these works has focused on the conditions under which the distribution of the cascade size follows a power-law. Interestingly, many of these variants can be mapped into a few model classes (Lorenz *et al.*, 2009).

Epidemic spreading and virus contagion models in the spirit of the famous Susceptible-Infected-Susceptible (SIS) model, can also be seen as a dynamic process with important formal similarities to financial contagion. A crucial result from the investigation of the SIS model is that scale-free networks behave markedly different from random graphs since the epidemic threshold of the infection rate tends to zero for large network size (Pastor-Satorras and Vespignani, 2001), meaning that no matter how small the rate of infection is there will always remain a significant fraction of infected nodes in the population.

Notice that in most cascade models based on the mechanism of load redistribution, adding links in the network tends to dilute the effect of a failure on the neighbors, meaning that a more dense network is also more robust against cascades. On the contrary, in contagion models, more links tend to help propagating failures more effectively so that a more dense network is more fragile. An important lesson for financial contagion follows from this consideration. In financial networks, both effects are present: On the one hand, links allow agents to diversify risk. On the other hand, agents with many links tend also to import distress from others (Stiglitz, 2010) and are exposed to amplification effects such as bank runs or trend reinforcement (Battiston *et al.*, 2012b). Thus it is highly misleading to transpose in financial networks the results from the literature on cascading models in other fields without realizing what mechanisms are at work from a dynamical point of view. Usually cascade models and contagion have been studied mostly in separate settings. Instead, they need to be taken into account simultaneously in order to understand the role of network topology as was recently shown in (Roukny *et al.*, 2013).

1.3 Issues on Financial Contagion and Networks

In economics, there is an established literature on default cascades that started from the realization that when financial institutions are connected in a network of liabilities and claims it becomes non-trivial to find out how much each owes to each other in case some of them default. The pioneering work of Eisenberg and Noe (2001) has formalized the problem in terms of a «fictitious default sequence» based on a fixed point approach. Essentially, given the full knowledge of the balance sheets in the bank networks, one or more banks are shocked in the beginning and new defaults are determined recursively together with the new values of the claims of those banks who survive. In principle, the total loss induced to the system by the default of a specific bank, provides also an estimation of how systemically important that bank is. A number of works have applied or extended this framework in a number of theoretical and empirical contexts and this works represent the state-of-the-art in stress-tests carried out at central banks (Cont *et al.*, 2011; Elsinger *et al.*, 2006; Gai and Kapadia, 2010; Mistrulli, 2011).

There are some problem with the traditional approach of default cascades. Typically, the exposure of a bank to another single bank is smaller than its equity. This is the result of regulations that over the years, even before the 2008 crisis were aimed at containing domino effects. This means that the default of one single bank does not cause any other default and the stress test indicates that the banking system is robust. There is a need to carry out stress tests that incorporate additional effects so to better explain what we observe empirically.

The first problem with the traditional approach is that in reality, when a bank faces a loss due to the default of a counterparty this may trigger additional losses, due for instance to the presence of short-term creditors who may decide to run on the bank. Section 2 will describe how this can be captured in a simple model and what insights we gain. In particular, this model delivers an interesting perspective regarding an important question, namely about what would be the optimal architecture of financial systems. For instance, several empirical studies show that financial networks are highly heterogenous in the number of contracts that each bank engages in – the «degree»– (Boss *et al.*, 2004; Iori, De Masi, Precup, Gabbi and Caldarelli, 2008; Soramäki *et al.*, 2007) and it is not clear how they could be made more resilient.

The work of Roukny *et al.* (2013) adopts the model developed in Battiston *et al.* (2012a) and carries out a comprehensive study of the interplay of the main drivers of systemic cascades: (1) network topology, (2) individual banks' capital ratios, (3) market illiquidity and (4) centrality of the banks initially shocked. Notice that such an extensive analysis is not trivial from a computation point of view. In this respect, the challenge is to find the right balance: the default cascading dynamics is simple enough to allow to run very extensive simulations on a variety of scenarios. At the same time, the model is derived from basic facts of banks' balance sheets and very much in line with those used in the stress tests (Cont *et al.*, 2011; Elsinger *et al.*, 2006; Gai and Kapadia, 2010; Mistrulli, 2011).

The results show that, in general, the architecture of the network (the «topology») does not matter when the asset market is liquid. In contrast, it matters a great deal when the market is illiquid. It is also not true that certain topologies are always superior to others. In particular, the so-called scale-free networks (see Appendix) can be both more robust and more fragile than homogeneous architectures. This finding has profound policy implications. It means that the optimal architecture depends on the level of market liquidity and suggests that regulators should be aware of the topology they are confronted with, before making decisions regarding liquidity injections. An illustration of what insights policy makers could obtain using live information from interbank markets, is obtained by running the model on the historical data of an electronic interbank market (the e-mid) from 1999 to 2011. From there one can see, with certain caveats, what would have been the effect on systemic risk of liquidity interventions in a range of scenarios regarding capital ratios and market illiquidity. Overall, this finding suggests that regulators could broaden the focus of their attention from capital buffers and bank size to the other dimensions that play a crucial role for resilience.

The second problem is that defaults are rare and not the only events that matter for the spreading of distress. Even if the obligor of a loan does not default, the fact that its equity is eroded implies that the market value of its obligations decreases. Because they are held by the counterparties there will be an effect, in turn, on their equity as well. There is no agreement on how we should quantify precisely the extent of this devaluation, which makes it difficult to develop a fully fledged model of this phenomenon. Yet, the effect is there and it may be substantial. In Section 3, we describe how this can be captured in an indicator to assess the systemic impact – DebtRank – of institutions even when no bank goes in default.

However, it is important to be able to assess the systemic importance of institutions not only in terms of contagion, but also in terms of liquidity provision. In Section 4, we see how, again, importance is only weakly correlated to size and other aspects have to be taken into account.

2 Default Cascades With Bank Runs

The model. In the model of default cascades with bank runs, the economy consists of N banks with the following balance sheet structure. The assets of each bank i include interbank assets A_i^I (i.e. mid and long-term investments in obligations of other banks) and external assets A_i^E (assets not directly related to any bank in the system). Similarly, liabilities include interbank liabilities and external liabilities. The external assets include short-term and thus liquid assets A_i^{ES} and less liquid ones, denoted as A_i^{EM}. The latter can be liquidated but, potentially, at a loss that depends on how illiquid is the market for those assets at the moment of the sale. Liabilities include interbank liabilities that are assumed to be mid-long term, and external liabilities, denoted as L_i^{ES} that instead are short-term and we assume are owed to creditors external to the banking system under focus.

Following a well-established approach (Eisenberg and Noe, 2001), we define as default of bank i at time t, the event of the equity of bank i becoming negative. We are interested in investigating how the number of banks defaulting in the system depends on the structure of the balance sheet interlock among banks as well as on the liquidity of the market of external assets. In the following, it turns out that it is mathematically convenient to focus on a capital ratio that measures the equity of bank i relative to its total interbanks assets, defined as $\eta_i = \dfrac{A_i - L_i}{A_i^I}$, where A_i and L_i are the total assets and liabilities of bank i.

We consider the usual mechanism by which a bank faces losses due to the default of some of its borrowers. As a benchmark, we consider the case of zero recovery rate, although this condition can be easily relaxed in the model. In addition, after suffering from losses due to the default of some borrowers, the short-term creditors of the bank may decide to run on their loans and refuse to roll over the short-term funding. As a result, bank i sells the assets necessary to pay back those liabilities. First, the bank sells the liquid ones. If needed it sells also parts of the less liquid ones. Depending on how illiquid the market, is in order to sell the latter assets, the bank is forced to sell them below the market price («fire-selling») incurring in additional losses.

The quantities that matter in the above sequence of events are the following: the difference $L_i^S - A_i^S$ between the amount that bank i has to repay and the amount that can be liquidated promptly; the ratio q between the market price for the less liquid assets and the fire selling price at which those assets have to be liquidated in order to find buyer. As a result, the parameter $b_i = (q-1)\dfrac{L_i^S - A_i^S}{A_i^L}$ represents the loss incurred by bank i in the process, measured in relative terms with respect to its total interbank assets.

The sequence of events in the model is as follows. There is a number of initial defaults that can, in turn, induce the default of others and the process stops when no more default events are observed. At the end, the size of the cascade, i.e., the total amount of defaulted agents, is recorded. The default is determined by the equity of a bank becoming negative. The equity of each bank may decrease over time as a result of two mechanisms. First, bank i faces the default of a counterparty, which implies that the loss of the corresponding asset A_{ij}, while liabilities of i remain the same. Second, bank i incurs a credit run from its short-term external creditors whenever the number of failures among the counterparties of i, relatively to the system size, raises above a certain threshold that depends on bank's *i* capital ratio. Formally, the condition reads as $\frac{k_{fi}(t)}{N} > \eta_i(0)/\gamma$, where γ is a parameter that measures the sensitivity of the external creditors. Notice that the bank run could be modeled as a game among the short-term creditors in line with an established stream of works. However, here we model it in reduced form in light of the fact that the macroscopic behavior of the default cascade depends on the conditions that trigger the bank run and not so much on the way the bank run unfolds.

In this model, the size of the cascade, i.e., the total amount of defaulted banks, can be computed analytically under some approximation regarding the structure of the network and correlations across defaults in the neighborhood of each bank (Battiston *et al.*, 2012a). Those analytical results are confirmed by simulations and extended to the case of heterogeneous networks (Roukny *et al.*, 2013). Here we summarize the results that are most relevant for the policy debate on capital ratio requirements.

Even in a minimal model, as the one considered here, if we want to investigate the resilience of different network architectures, there are several degrees of freedom to consider. In order to reduce the number of scenarios that have to be examined, we make the following choices.

- We vary the type of shock: randomly vs targeted. In the first case, the banks that are shocked in the beginning are chosen at random while in the second they are chosen based on their degree of centrality.
- We vary the correlation between capital ratio and degree. In case of no correlation, the structure of the balance sheet is assigned across banks in a way that the capital ratio is independent of the number of contracts the bank holds. In the case of positive correlation, the higher is the degree of the bank the higher is the capital ratio, and *vice versa* in the case of negative correlation.
- We vary the degree distribution by comparing scale-free networks with random graphs and regular graphs (see Appendix).
- We vary the market illiquidity by means of the parameter *b* defined above.

A combination of the above choices is indicated in the following as a «scenario». In each scenario then, we study the cascade size (that is the fraction of banks that default in the end) as a function of the average out-degree *k* in the network and the average value *m* of initial individual capital ratio. The initial capital ratio $\eta_i(0)$ is allocated across banks according to a Gaussian probability distribution with mean μ and variance σ. Negative values of $\eta_i(0)$ imply that the corresponding banks are in default at the beginning. We refer to these as *endogenous shocks*. In addition, a fraction y_0 of banks is additionally set to default. These are referred to as *exogenous shocks*.

Experimental Set-Up. The initial value of the capital ratio $\eta_i(0)$ is assigned across banks as a random variable according to a Gaussian distribution with mean μ and variance $\sigma_\rho^2 = \sigma^2/k$ where μ and σ are exogenous parameters for the experiments. The relationship between σ_ρ and the average number of connections of the system *k* reflects the assumption that a larger number of credit counterparties leads to a smaller variance in the return of the credit portfolio of each agent and, thus, in the individual robustness. Capital ratio is either assigned randomly or with respect to the bank's degree, depending on the scenario implemented. As a gaussian distribution of robustness can produce negative values, agents starting with $\eta_i(0) < 0$ are considered to be *endogenously* set in default. In addition, *exogenous shocks* are introduced by putting some agents with $\eta_i(0) > 0$ into default according to an external parameter y_0. For a discussion of the choice of the parameter values see Battiston *et al.* (2012a). Simulations are carried out with a population of 1,000 banks. Every simulation generates a network realization and runs the cascading process on such realization. For a given topology, we run 1,000 simulations for each pair of parameter values. For each pair of values of (b, m) and (k, m), we record the average cascade size and the standard error. We then determine the curve representing the *frontier* in the parameter space between the region where large cascades occur and the region where small cascades occur.

Formally, our network is defined as directed and weighted. The direction of a link goes from the lender to the borrower and the weight of a link corresponds to the amount at stake from the lender's perspective. We assume that each bank holds an equally weighted portfolio i.e., the relative exposure to each borrower is equal and amounts to $1/k_i$, where k_i is agent i's number of borrowers. Since networks are directed, two different distributions should be distinguished. The out-degree distribution characterizes the lending behavior, while the in-degree distribution describes the borrowing behavior. Here we report on the case in which in and out degree are correlated, but other cases have been studied too.

Empirical dataset of the e-MID interbank market. For the simulations on empirical data, we use a collection of daily snapshots of the Italian interbank money market originally provided by the Italian electronic Market for Inter-bank Deposits from January 1999 to December 2011. The data is maintained by e-MID S.p.A., Societa Interbancaria per l'Automazione, Milan, Italy and we refer to it as e-MID in the text. After aggregating the lending relations on a monthly basis, we extract the structures of interaction between banks. Hence, we use a collection of empirical networks describing the chronological evolution of the successive topologies that the Italian interbank money market went through from the beginning of 1999 up to the end of 2011.

The results for the cascade size are illustrated in the form of phase diagrams where each curve in the diagram represents, for a given network topology, the *systemic risk frontier* between high systemic risk (very large cascades, region in color) and low systemic risk (very small cascades, region in white) in the parameter space of market illiquidity b and average capital ratio m. As we can see, moving upwards and leftwards corresponds to moving to a system with higher capital ratios and higher liquidity. A different color is associated with each topology so that the diagrams allow to see how network topologies are affected by illiquidity and what level of average capital ratio would be needed in order to move the banking system into the safe region.

Figure 1 left (right) refers to the case of positive (negative) correlation between capital ratios and centrality degree. We can see that the systemic risk frontier for the scale-free topology is below the others meaning that this topology is more robust against random shocks in case higher capital ratios are allocated to more central banks. The opposite occurs when more central banks have smaller capital ratios. However, the differences are important only when the market illiquidity is sufficiently high (b large than 0.3). When the shocks are targeted towards the most central banks, the fragility of scale-free networks

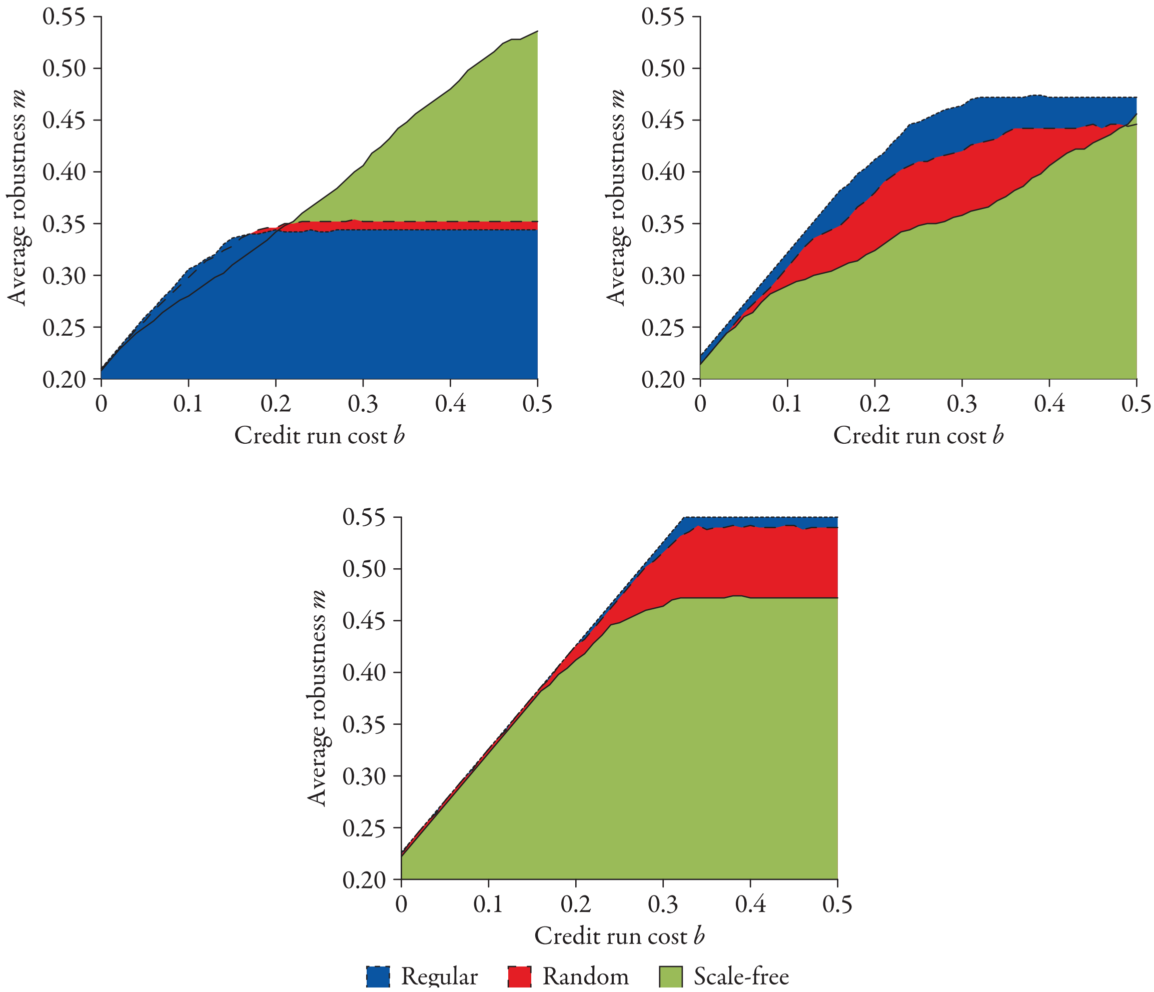

Figure 1: Frontier of large cascades in the space (b, m) representing average capital ratio across banks and market illiquidity. (left) Random exogenous defaults and positive corre- lation between degree and individual robustness. (right) Random exogenous defaults and negative correlation between degree and individual robustness, Networks have an average degree, $\bar{k} = 20$. Other parameters values are: $\gamma_s = \gamma 10^{-3}$, $\gamma_s = 0.13$ $\sigma = 0.3, y_0 = 0.04$.

is exacerbated for any level of market liquidity (results not shown). Interestingly, the latter result is in line with classical works in complex networks using a different notion of robustness, i.e. measured in terms of the size of the largest connected component that remains after the shock.

We can also apply the same approach to the empirical data of a specific interbank market. Notice that this is not done as a validation of the default cascade model (this would require to follow chains of default events across the network). The dataset consists of a collection of monthly snapshots of the network of the so-called e-MID interbank money market. Links represent lending flows among banks aggregated at the time scale of a month. The data spans the period between January 1999 and December 2011. We analyze the evolution of systemic risk frontiers for the months of January of each year between 1999 and 2011.

As shown in Figure 2, the frontiers of the periods 1999-2008 (a smooth and linear dependence on illiquidity b) can be separated from those of the periods 2009-2011

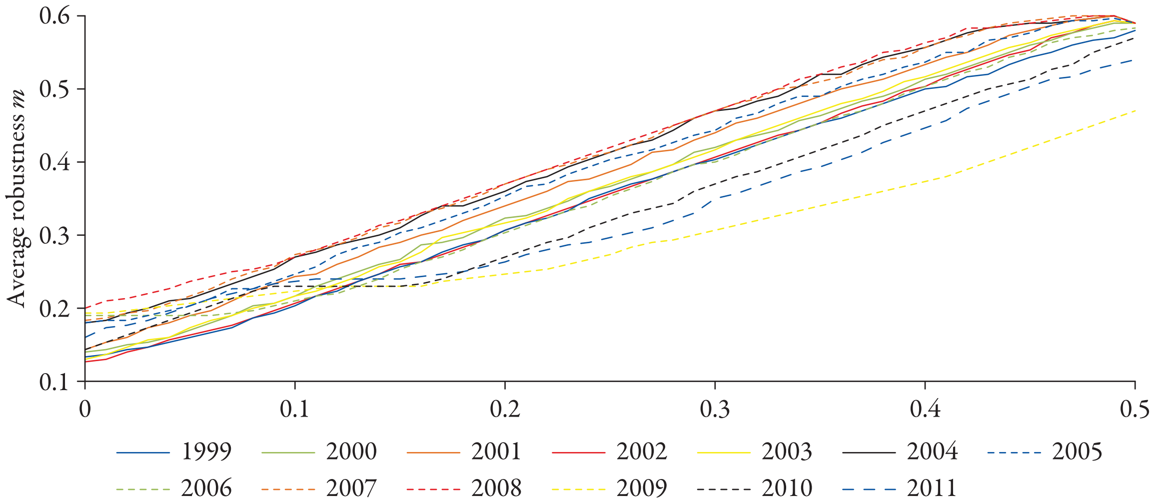

Figure 2: Frontier of large cascades evolution of the e-MID market in the period between January 1999 and January 2011 under random exogenous defaults and random individual robustness distribution. Impact of illiquidity on the structure of January of each year, $\sigma = 0.3$, $y_0 = 0.04$, $\gamma_s = 0.13$. For convenience, we use $\gamma_s = \gamma 10^{-3}$.

(curves are located at lower values of individual robustness m and tend to be less sensitive to illiquidity). The years of 2007 and 2008 correspond to the highest systemic risk frontier while the year of 2009 corresponds to the lowest frontier (i.e. less systemic risk). In order to understand how to interpret this result, it is important to recall that the period between January 2008 and January 2009 corresponds to the post-Lehman Brothers era, marked by (*i*) an important rise of interbank rates for all major currencies and (*ii*) a takeover of central banks to provide liquidity and guarantees to banks. As banks became more reluctant to engage in credit exposures with other banks and started trading with the central bank (that is not present in our dataset), default cascades across the interbank market became obviously much less likely to be triggered. This explains the sudden drop of the 2009 frontier with respect to the previous years and its smaller sensitivity to illiquidity (i.e. smaller slope). After 2009, banks slowly started to engage again in the interbank market making it more sensitive to illiquidity as shown by the 2010 and 2011 curves in Figure 2.

In particular, the case of 2009 provides insights on the impact of big provision policy guaranteed by the European Central Bank (ECB) at that time. Starting from the Fall of 2008, this action along with the important rise of interbank rates made banks less active in the interbank market. This, in turn, decreased the sensitivity of the market to illiquidity: in the presence of a smaller system in terms of both size and density, the amplification phenomenon depicted by our model loses its impact. In simpler terms, banks lend less to each other, thus reducing the impact of a credit run on any bank. Finally, we can imagine that this apparent benefit is not without drawbacks since the ECB is not recorded in our data: part of the previously captured risk has been transferred from the e-MID to the ECB. In light of the results from the synthetic simulations, introducing the ECB would indeed increase the heterogeneity of the underlying network. The case would become extreme: the ECB would appear as the *node of last resort* to avoid a system collapse.

These results illustrate how models of this type can be used to understand the systemic impact of a shock on an interbank market from a macro-prudential point of view, taking into account the networks for various levels of capital ratios and illiquidity. This can help central bankers in designing *ex ante* capital structure requirements and ex-post liquidity provisioning schemes.

3 DebtRank

3.1 Algorithm

DebtRank is a measure of the systemic impact of an institution on the others that was introduced in Battiston *et al.* (2012c). DebtRank of bank i, denoted as i, is a number measuring the fraction of the total economic value in the financial network that is potentially affected by the distress or the default of bank i. The method differs from the ones based on the default cascade dynamics described in Section 2. In those models, below the threshold no impact is propagated to the neighbors. Instead here in this model, the distress propagates even below the threshold of default.

In order to explain the functioning of the method, we introduce as in Section 2 a directed network in which the nodes represent institutions and the links represent financial dependencies. We denote the amount *invested* by i in the funding of j as A_{ij}. Thus, A is the weighted adjacency matrix of the investment network. The total value of the asset invested by i in funding activities is $A_i = \Sigma_l A_{il}$. We denote by E_i the tier 1 capital the capital of i, which works as a buffer of i against shocks Cont *et al.* (2010); Mistrulli (2011). Bank i defaults when $E_i \leq 0$.

It is important to notice that we are interested in the situation in which each banks' liabilities are accounted at their face value while assets are marked to market. The intuition that DebtRank aims to capture is that when, ceteris paribus, the equity of i decreases because of a shock, even if not to the point of inducing its default, the market value of the obligations of i decreases because bank i's distance to default is smaller or in other terms, bank i is less likely to meet its obligation at maturity. Moreover, because others have bank i's obligation in their balance sheet their equity decreases too and the distress propagates in the network.

The amount by which the market value should decrease is not trivial to determine. In fact, the new market value of bank i's obligation depends on bank i's probability of default and the recovery rate on its assets. The problem is that both these two quantities depend on the market value of the obligation of other banks that bank i has in its own portfolio. Currently, there is no model in the literature that allows to compute these quantities in a system context.

We can conjecture that the relation between losses on equity and losses on obligations is non-linear: indeed when small losses on equity should not be reflected in any loss on obligations, while big losses that almost deplete all the equity should all have the same effect. Because we do not have at the moment a theory for this or a systematic empirical stylized fact, modeling this non linearity would imply us to introduce new parameters. For the sake of parsimoniousness, in its current formulation DebtRank assumes the simplest relation between losses on equity and losses on obligations that is a relation of proportionality: we assume that if the market value of the obligation A_{ij} of i to j decreases, in relative terms, as much as the equity of i. So if the equity E_i of i decreases by, say, 20%, then the value of its obligation A_{ij} does too. In turn, the equity of j will also decrease, proportionally to the exposure of j to i. As a result, now the obligation of j is worth less and the effect propagates down to some other counterparty k. In future formulations, the relation equity-obligation could be modelled in a more realistic way. The linear case is however a paradigmatic case that is useful to test as the most obvious benchmark.

We can capture the mechanism just described by defining the impact of i on j as $W_{ij} = \min\{1, A_{ij}/E_j\}$. Thus, if the loss exceeds capital, the impact is 1. Notice that the matrix W is, in general, neither column-stochastic nor row-stochastic. We further take into account the economic value of the impact of i on j by multiplying the impact by the relative economic value of the node j, $v_j = A_j/\Sigma_l A_l$ (other proxies could be taken for v_j). The value of the impact of i on its neighbours is then $I_i = \Sigma_j W_{ij} v_j$, which measures the fraction of economic value in the network that is impacted by i directly.

We now want to take into account the impact of i on its indirect successors, that is, the nodes that can be reached from i and are at distance 2 or more. To this end, we introduce the following process. To each node we associate two state variables. h_i is a continuous variable with $h_i \in [0, 1]$. Instead, s_i is a discrete variable with 3 possible states, undistressed, distressed, inactive: $s_i \in \{U, D, I\}$. Denoting by S_f the set of nodes in distress at time 1, the initial conditions are: $h_i(1) = \psi \forall i \in S_f$; $h_i(1) = 0 \forall i \notin S_f$, and $s_i(1) = D$, $\forall i \in S_f$; $s_i(1) = U \forall i \notin S_f$. The parameter ψ measures the initial level of distress: $\psi \in [0, 1]$, with $\psi = 1$ meaning default. The dynamics is defined as follows,

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ji} h_j(t-1)\right\}, \quad \text{where } j \mid s_j(t-1) = D, \tag{1}$$

$$s_i(t) = \begin{cases} D & \text{if } \ h_i(t) > 0 \ \text{ and } \ s_i(t-1) \neq I \\ I & \text{if } \ s_i(t-1) = D \\ s_i(t-1) & \text{otherwise,} \end{cases} \tag{2}$$

for all i, where all variables h_i are first updated in parallel, followed by un update in parallel of all variables s_i. After a finite number of steps T the dynamics stops and all the nodes in the network are either in state U or I. The intuition is that a nodes goes into distress when a predecessor just went into distress and so on recursively. The fraction of propagated distress is given by the impact matrix W_{ij}. Because $W_{ij} \leq 1$ the longer the path from the node i initially into distress is and node j, the smaller indirect impact on j is. Notice that when a node goes in the D state, it will move to the I state one step later. This means that if there is a cycle of length 2 the node will not be able to propagate an impact on its successor more than once. This condition satisfies the requirement, mentioned earlier, of excluding the walks in which an edge is repeated. An illustration on a simple example is provided in Section 3.2.1.

The DebtRank of the set S_f is then defined as:

$$R = \sum_j h_j(T)v_j - \sum_j h_j(1)v_j, \tag{3}$$

where R measures the distressed *induced* in the system, excluding the initial distress. If S_f is a single node the DebtRank measures the systemic impact of the node on the network. In this case, it is of interest to set $\psi = 1$ and to see the impact of a defaulting node. If S_f is a set of nodes it can be interesting to compute the impact of a small shock on the group. Indeed, while it is trivial that the default of a large group would cause the default of the whole network, it is not trivial to anticipate the effect of a little distress acting on the whole group.

3.2 Applications of DebtRank

The largest borrowers of the FED's 2008 emergency program. DebtRank overcomes the problem that when evaluating the systemic importance of an institution in a stress-test it is difficult to see any effect at all on the other institutions if only default is taken into account. As we discussed in Section 2, one can introduce bank runs or other negative externalities that amplify the initial shock but that comes at the cost of making additional assumptions in the model. DebtRank is a complementary approach.

A first application of DebtRank on empirical data was carried out in Battiston *et al.* (2012c) using a synthetic dataset of interbank exposures reconstructed under various scenarios based on the mutual investments of banks in each other's equity. The work focuses on the international banks that were the largest borrowers from the FED's emergency program of 2008-2010. The use of synthetic exposure data is due to the fact that cross border exposures among large institutions are unknown, even to regulators, despite the size of the players. Moreover, the systemic impact of each institution is computed with respect to the others in the group and not with respect to the financial system as a whole. Therefore, the interest of the exercise is not so much in the value of the systemic impact but rather in the illustration of what could be done if the data were available. Some results are robust across the range of values for the rescaling of the mutual exposures. The first is that systemic impact and size are weakly correlated, especially in bad times, i.e. when balance sheets are weaker. The second, which is also related to the first result, is that in bad times smaller institutions can be as systemically important as the biggest ones.

Here, we report on a type of analysis that was not covered in Battiston *et al.* (2012c). Indeed, DebtRank allows us to compute not only the systemic impact of each institution but also the vulnerability of that institution to an indirect shock, that is, a shock hitting other institutions. This is measured by the final value of h_j in Eq. (1), under the assumption that all banks i are hit by a small shock.

The scatter plots in Figure 3 show the values of DebtRank versus vulnerability for the top 22 largest borrowers of the FED in two periods of time. The size of each bubble is proportional to the outstanding debt of the institution towards the FED while the

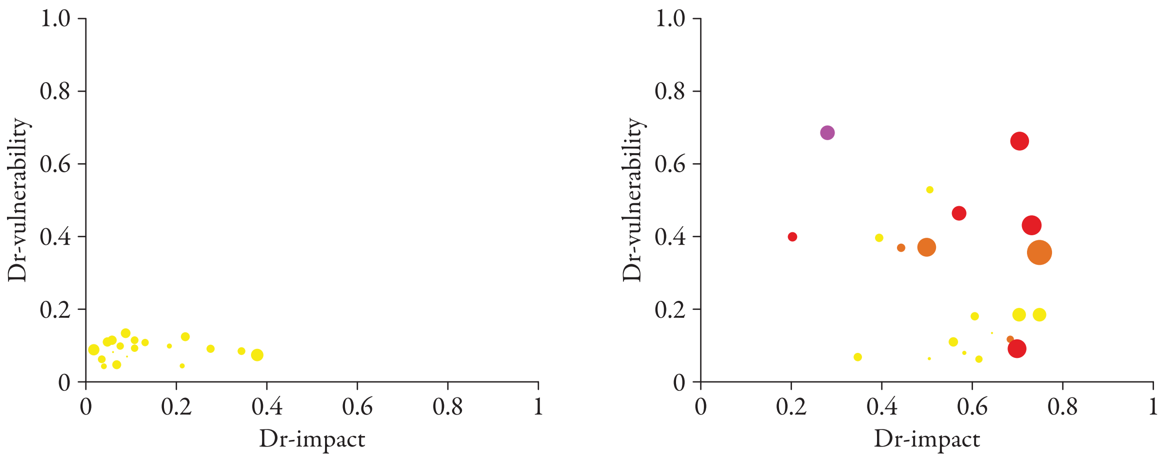

Figure 3: **DebtRank vs vulnerability**. Scatter plot of DebtRank versus vulnerability measured as the distress experienced by the institution as a result of a shock of 10% on the equity of all the other institutions in the network. The size of each bubble is proportional to the outstanding debt of the institution while the color reflects its fragility, defined as the ratio of debt over market capitalization in the given period, as in the previous section. Left) March 2008. Since the outstanding debt was very low or zero, most nodes appear small and have levels of DebtRank below 0.4, while the vulnerability values are small and comparable among each other. Right) March 2009. Many institutions have a DebtRank larger than 0.5, i.e. each can impact, alone, the majority of the economic value in the network. The outstanding debt in this period is close to the peak for all the institutions, as reflected by the size of the bubbles. Notice, also a higher fragility, most bubbles are dark red, although with some heterogeneity.

color reflects its fragility, measured as the ratio of debt towards the FED over market capitalization in the given period.

In March 2008 (left panel), the outstanding debt was very low or zero, hence most nodes appear small and have levels of DebtRank below 0.4, while the vulnerability values are small and comparable among each other. In March 2009 (right panel), several institutions have a DebtRank larger than 0.5, i.e. each can impact, alone, the majority of the also economic value in the network. Moreover, some have a high value of vulnerability, implying that they would go into distress if there is a shock to the counterparties. In this period institutions have also a much larger debt towards the FED and a higher fragility, since most bubbles are dark red. Highly fragile institutions that also have high impact would be eligible for a more strict follow-up by banking supervision.

As we describe later on in this section, a similar analysis has been conducted by other authors on the Brazilian interbank market (Tabak, Souza and Guerra, 2013).

The interbank exposure data set of Bank of Italy and Central Bank of Brazil. The DebtRank algorithm has been applied in some central banks to the analysis of systemically important financial institutions in national interbank markets. We summarize here some of the findings that are relevant to the present discussion.

The DebtRank analysis of the Italian interbank market reveals that systemic risk in such part of the financial market has decreased in the period 2008-2012 (Battiston, di Iasio and Infante, 2013). This can be interpreted as a decline of the systemic risk in the market conditional to a shock to one or more banks. It turns out that this decline can be explained by the decrease in the number and in the volume of contracts that has occurred in the same period in the unsecured interbank market. In principle, the decline of systemic risk could also be due to the concurrent decrease of banks' leverage as a result of the process of recapitalization of their balance sheets. However, the analysis in Battiston, di Iasio and Infante (2013) tries to disentangle the two mechanisms by constructing a dataset where the exposures are those from 2008 while the capital is taken from 2012, or viceversa. In this way, looking at hypothetical situations the authors find that the effect of the decrease in network density dominates the effect of recapitalization. Indeed, the decrease of the network density is related to the onset of the sovereign bond crisis of 2010, which caused a drop in the amount of wholesale funds reaching the domestic Italian interbank market from international institutions. This first finding highlights the applicability of DebtRank to investigate the effect of recapitalization policies in real interbank markets. It also points to the importance of multi-level financial networks since interlinkages across different markets may shift systemic risk from one market to another.

As a second relevant finding, the empirical analysis of the italian data confirm the results found also on the FED data that while DebtRank and bank's size tend to be positively correlated, in general the relation is non linear and banks with similar size may show very different DebtRank values. In particular, in the Italian market we observe some medium-size institutions that reach significant values of DebtRank because they act as liquidity hubs for small, co-operative banks.

The analysis of the Brazilian interbank market (Tabak, Souza and Guerra, 2013) uses DebtRank to carry out several exercises. They first identify institutions that have high impact and are fragile at the same time. Institutions with the highest impact and participation in the interbank market are found to be not very fragile as measured by their leverage. Nevertheless, there are institutions with high impact and high participation in the market that are fragile, with significant leverages.

The second analysis combines the impact of an institution as estimated by DebtRank with the probability that such an institution defaults, as given by the Merton formula. The result is an expected impact of that institution. The default probabilities are computed for each individual bank independently, neglecting joint default probabilities and conditional default probabilities. Large institutions tend to present lower expected impact due to their smaller default probability.

A third analysis looks at groups of systemically important institutions. One aspect is whether the composition of the top 10 institutions ranked by DebtRank is stable. Indeed, during the 1-year period, the institutions that enter this group are only 14 overall while 5 of them belong to the group at all times. Further, in order to select institutions that could more likely default together, the authors look at the default probabilities correlation matrix computed for the top 40 DebtRank institutions and compute the minimum spanning tree to extract those that are closest. The underlying idea is that common factors could increase the probability of joint defaults. The authors extract what can be seen as a proxy for the groups joint default probability and compute then the systemic impact, were the institutions in the group to default simultaneously.

DebtRank has also been used in an agent-based model by some authors (Thurner and Poledna, 2013) to investigate how systemic risk in financial networks could be reduced by increasing transparency. The DebtRank of individual banks is made visible to the other banks in the model and a rule is imposed to reduce interbank borrowing from systemically risky nodes.

4. Controllability

The final step in the possible actions related to the issue of systemic risk is to understand if regulators may be provided with some instrument to drive the dynamics of financial systems towards any desired state. One of the most interesting approaches that has appeared recently has to do with the concept of controllability. Structural controllability in this context has to do with the possibility to drive the dynamics of a specified system into a particular state. In order to have a mathematically well defined theory we need to define a state function on the vertices of our system. That is, on every vertex i we have a variable x_i so that the n-ple $\vec{x} = (x_1, x_2 \dots x_n)$ represents the state of the whole system. We assume a linear dynamics of the form $dx(t)/dt = Ax(t) + Cu(t)$ where $x_{i = 1, \dots, n}$ is the state variable for the n nodes, A is an $n \times n$ «influence» matrix representing the way every node is influenced by others. The components of the vector u correspond to external functions that are applied to a subset of n_D nodes and C is an $n \times n$ «control» matrix of external weights.

The mathematical basis of structural controllability has been recently extended to the case of networks (Liu *et al.*, 2011). The idea is that the whole network can be «controlled» by acting on a specific subset of nodes, which counterintuitively, happen often no be the most connected ones. The mathematical passage necessary to understand the topic is to look for the maximum matching in the oriented graph. In steps it works as follows:

- The first step is to compute the matching edges A matching in a graph $G = (V, E)$ is a set of edges, none of which have a common end (vertex).
- The second step is to collect the endvertices of the matching edges (those with arrows pointing to them) that form the matching vertices.
- The nodes untouched form the set of drivers on which we can «economically» to change the state of the system.

The last passage is ensured by the so-called minimum input theorem stating that the minimum number of nodes to be controlled (drivers) corresponds to the number of uncovered nodes in a maximum matching of the corresponding graph. A typical case is shown in Figure 4.

Despite the technical difficulties (typically there is a huge number of possible configurations all with different drivers set), the possibility to drive a financial system towards a desired state is very tempting. Indeed, if this were true we could in principle give to regulators an instrument to drive all the banks into this situation, The specific case study considered (Delpini *et al.*, 2013) assessed the controllability of interbank money markets and in particular the Italian electronic trading system (e-MID), for which a time series of micro data is available. The data presented in the analysis is composed of 2,750 daily

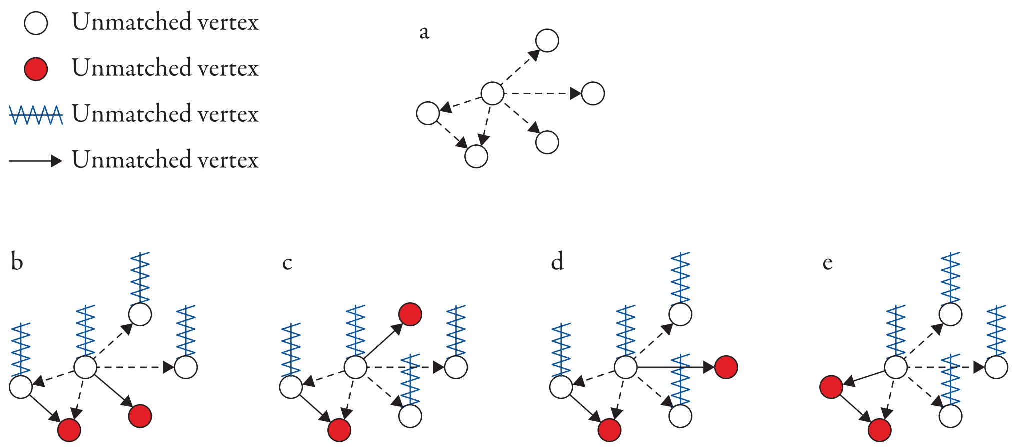

Figure 4: The concept of controllability. Starting from network (a) one first finds the matching edges (4 different choices in b, c, d, e) and from those the matching vertices. The vertices unmatched needs to be driven from external regulation if we want to drive the system towards a desired state.

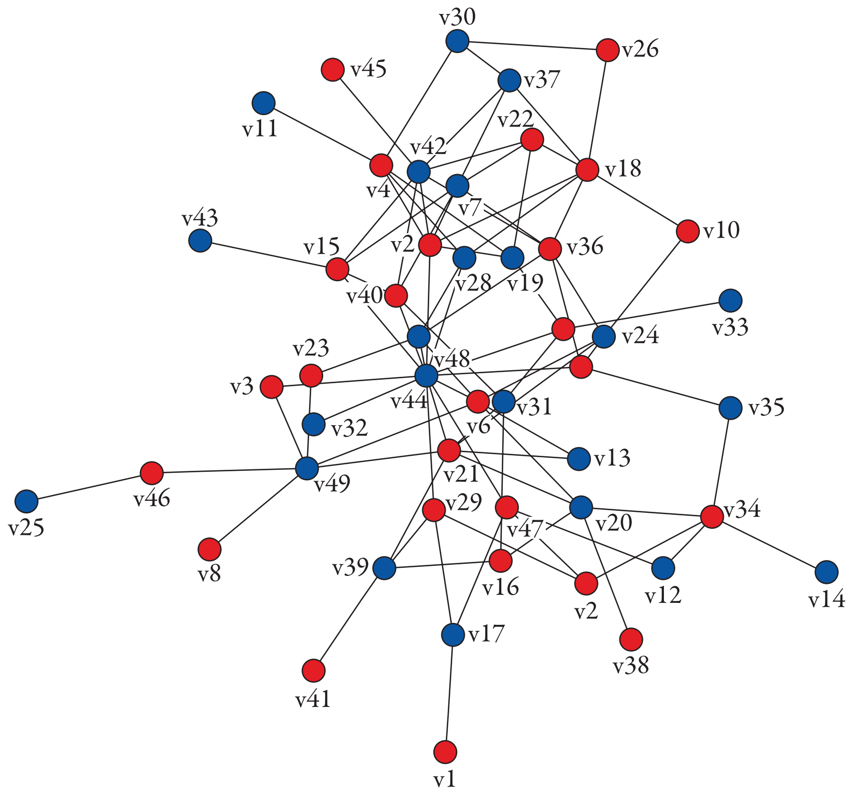

Figure 5: A sample snapshot of the daily interbank lending network. Figure from Delpini *et al.* (2013).

snapshots of the Italian interbank money market. Data spans from January 4th, 1999 to September 30th, 2009 and for the most part, the transactions correspond to overnight exchanges of deposits among banks (Fig. 5).

In this study the state variable x_i is the level of funding that bank i provides to the others. Similarly, x_i depends on the funding that the same bank gets from its neighbours. This assumes that banks influence each others through «funding contagion» and that the influence of bank i on j is somewhat proportional to the funding provided by i to

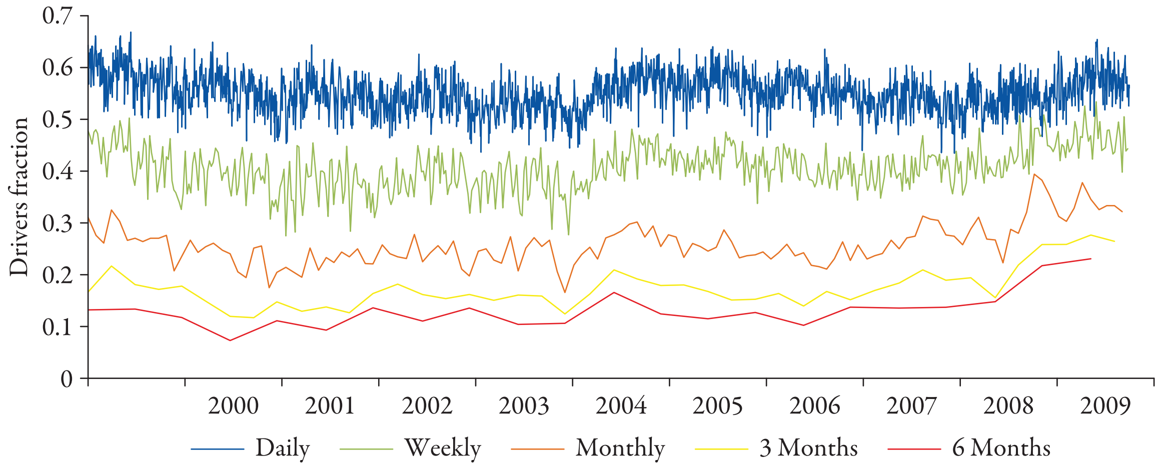

Figure 6: Time evolution of the fraction of controlling banks: the controllability depends crucially on the time resolution. At the monthly aggregation scale less than 40% of the banks drive the system. Figure from Delpini *et al.* (2013).

j. In this mapping, the external «control» corresponds to liquidity interventions of central banks in individual institutions of the network. Technically, these interventions are liquidity refinancing or, in principle, dedicated credit facilities. While central banks cannot actually enforce banks to lend, they can nonetheless provide liquidity to key-role banks in the market on a much larger basis than they need. In practice, this could be expected to induce very liquid banks to effectively provide liquidity to the other players.

Following the above reasoning, the larger the weight of a link, the larger the impact of *i* upon *j* and, intuitively, maximising the sum of weights (all weights are positive in our case) would mean maximising the sum of the impacts. As mentioned above, even by considering a simple step of the network relative to a single day, we deal with a graph where the number of maximum matchings becomes really large. In this respect, introducing the constraint of maximizing the sum of the impacts reduces significantly the number of configurations. In particular, we can attach a weight to every transaction between banks making the hypothesis that cash flows between banks are a good proxy of the influence between the banks. The maximum matching is then defined as the matching with maximum weight. A typical case of driver nodes is shown in Figure 4 where the white vertices represent driver nodes. Remarkably, despite the large number of similar configurations, few configurations result in driver nodes being the hubs of the network. Indeed, if we analyse the networks constructed from the e-mid data, we find that the average degree of the driver nodes $\langle k_D \rangle$ is systematically smaller than the average degree of the nodes in the network. This finding holds regardless of the aggregation level, implying that in the bank network the drivers are typically not the hubs (Fig. 6).

At this point, one could take the fraction of network drivers n_D as an indicator of control efficiency and robustness. A large value of n_D would indicate that the interbank system is problematic from a control perspective, since many banks would be responsible for variations of the state of the whole network. Conversely, regulators and policy makers would appreciate small values of n_D. Another interesting finding from the analysis of the e-mid data is that looking at the daily evolution of the lending network would be misleading. Indeed, at such time scale the number of drivers is quite high (about 60%).

This is because more than half of the banks are drivers. However, our study reveals that the fraction of drivers decays according to a power law as the aggregation scale gets larger as shown in Figure 6.

However, the n_D values of drivers, on average, decrease monotonically as a function of the aggregation scale and at the scale of the month the network appears to be «controlled» by less than 30% of the banks. With values of n_D about 30% it becomes more feasible for the regulator to implement targeted interventions on the set of driver banks. Incidentally, notice that in other systems such as the WWW or the Internet typically a higher fraction of drivers has been found, e.g. $n_D > 50\%$.

5 Discussion

In this paper we have discussed some of the insights that network models bring to the investigation of financial systems.

The insight from models of default cascades in presence of illiquidity is that looking at capital ratios only is not enough, but also looking at topology only is not enough. We actually need to look at the interplay of topology, liquidity and capital buffers. When liquidity is high, the architecture of the market does not play any role: different networks provide similar stability profiles. When liquidity is low, it makes a big difference whether the network is very heterogeneous (scale-free) or homogenous (regular graph). However, the distribution of capital buffers (e.g. whether the hubs are more capitalized or less capitalized) can reverse the results. Given the current context where banking regulation remains mainly at the individual level, those results show that the way claims and liabilities are intertwined within financial markets, creating highly complex financial networks, should not be neglected. Therefore, the findings suggest the need for regulators and policy makers to acquire a sufficiently detailed map, not only of the individual balance sheets but also of the structure of mutual exposures and market conditions they are confronted with in order to make better decisions.

The insight from the various works on DebtRank are that there is more to systemic importance than size and position in the network, but the condition of the balance sheets of counterparties is crucial to determine the impact that an institution can cause to the system.

Finally, the main insight from the work on controllability is that the top «controlling» institutions are often not the hubs in the network nor the major lenders. Moreover, the set of such institutions, which are systemically important for the liquidity, may vary with the time scale that we look at. This implies that effective regulatory supervision cannot simply focus on the biggest banks and that the notion of systemically important bank should take into account the time scale of the transactions.

Most macro-prudential policies for financial stability focus on individual bank ratios such leverage or capital adequacy ratios or equity ratios. Then, in terms of assessing the systemic importance of the various institutions, most of the attention has been on bank size. The dimension of interconnectedness (meant as amount of exposures on the interbank market) has been included (along with others) in the IMF/BIS/FSB report submitted to the G20 Finance Ministers and central bank Governors in October 2009. Moreover, the Basel Committee on Banking Supervision (2013) has recently suggested to include the dimension of complexity, as a measure of the cost of resolving the bank, which depends on the amount of notional OTC derivatives held by banks.

In the context of such debate, three interrelated dimensions play a major role in the analyses presented earlier: interconnectedness, complexity and correlation.

As we have seen earlier, in the model of default cascades higher interconnectedness leads to higher systemic risk when coupled with illiquidity and low capital buffers. The DebtRank method also shows that a higher interconnectedness among banks increases the systemic impact of each bank over the others. In particular, if a bank keeps its amount of exposures and diversifies them over a larger number of counterparties, this is beneficial for the individual bank as it reduces the loss from any single counterparty. However, such diversification increases the chances that the bank will act as channel to spread the distress from a shocked bank to a third one. Overall, a fully connected network spreads around more distress than a sparse network.

More in general, besides the interconnectedness arising from the interbank lending, it is useful to think of the interdependence of balance sheets and payoffs of banks arising from various financial instruments. A general insight from the study of financial networks is that interdependence is a source of systemic risk, as soon as positive feedbacks are present in the system (Battiston *et al.*, 2012a). Now, positive feedbacks are very often present in financial markets, either visible or latent. An example is the procyclical spiral fire-selling-asset devaluation, which can be triggered by a change in agents' expectations on the future value of that asset. Clearly this can also be seen as an effect the potential illiquidity of the market for assets. Another example is the fact that the very reaction of creditors (e.g. tightening credit conditions) to a first deterioration of an obligor's equity ratio, is likely to induce its further deterioration. This is also a manifestation of a positive feedback. In the natural sciences, a system where positive feedbacks prevail is prototypical of a unstable system. If its units are also highly interdependent it is immediately recognized as prone to systemic risk.

The complexity of banks may well be seen as to contribute to their interdependence, due to the OTC derivatives contracts that a bank establishes with others. The argument that these contracts help to diversify and reduce risk is controversial (Battiston *et al.*, 2013). While the dimension of complexity did not appear directly in the models presented above, the complexity of financial instruments is likely to contribute to the potential illiquidity of the market. Indeed when players start questioning the value of an asset, its complexity is not of help in making counterparties willing to buy it. Another problem of complexity is that it makes room for information asymmetries that in bad times can be exploited by market players as an argument for being too complex-to-fail (Battiston *et al.*, 2013). This exacerbates the effect of the findings from the DebtRank method where in times of low capitalization all banks become systemically important.

Finally, the correlation of banks' behavior is another important dimension that indirectly contributes to the potential for market illiquidity. Clearly, the more banks have made correlated choices in their portfolio, the stronger will be the effects when they all try to fire-sell the same type of asset.

Overall, the findings discussed in this paper and in other cited works in financial networks (e.g., Battiston *et al.*, 2013; Gai *et al.*, 2011), suggest that in order to contain systemic risk, besides maintaining capital ratios, it is necessary (but maybe not sufficient) to decrease *simultaneously* the interrelated dimensions of *interconnectedness*, *complexity* and *correlation*. It remains an open and question how to achieve this objective. For instance, it is challenging to design mechanisms to contain interconnectedness and correlation. However, in our view, the various proposals to reform the structure of banks and the architecture of the financial system should be first tested against their ability to deliver progress in this direction. As an example, splitting banks in commercial and investment arms does not, per se, prevent the investment arms of various banks to remain too much connected, complex and correlated. Even if balance sheets of the two arms are virtually separated, once this compartment of the financial system gets in trouble, the distress will propagate to the commercial arms by some other channel. As an urgent future avenue of research, we advocate a thorough comparison of different proposals with respect to those three dimensions as a prerequisite for a more informed debate.

6 Appendix

6.1 Network Glossary

We report a list of the definitions that are relevant to the paper.

Consider a simplified banking system composed of n banks and m assets.

- In our context, an interbank **network** G is the pair (N, E) consisting of the set of **nodes** or **vertexes** $N(G) = 1, ..., n$ representing banks, and a set of edges $E(G)$, or links, representing financial contracts among banks.
- If the network is directed, an **edge** is an ordered pair of nodes (i, j) representing in our context that bank i lends to banks j. A weight A_{ij} can be associated with the edge indicating, for instance, the nominal value of the contract. If the network is undirected the order of the pair is not relevant, $(i, j) = (j, i)$ indicate the same edge.
- The **adjacency matrix.** A of size $n \times n$ where n is the **order** of the graph is defined as follows. The element A_{ij} is not zero if an edge goes from i to j. The component i, j of A_{ij} is the weight of the edge.
- A network is said to be *bipartite* if the nodes can be grouped in two classes such that no edge exists between any two nodes of the same class. In our context the network of banks and assets is a bipartite network.
- The **neighborhood** of a node i is the set $N_i = jN : ijE$.
- The (connectivity) **out-degree**, or out-degree of a node i in G, denoted as k_i, is the number of edges outgoing from i. Similarly we can define the **in-degree**. If non specified we mean the **total degree** or the degree in the case edges are undirected.
- **Hub** *A* vertex with large degree.
- A **path** between two nodes i_1 and i_k is a sequence of nodes $(i_1, i_2, ..., i_k)$ such that (i_1, i_2), (i_2, i_3), (i_{k1}, i_k). In other words, it is a set of edges that goes from i_1 to i_k.
- The **distance** between two vertexes is the number of edges in a shortest path connecting them.

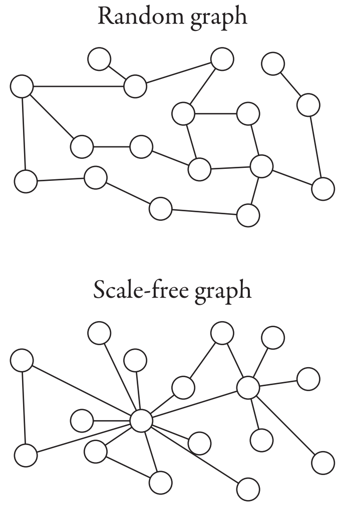

Figure 7: Above a Random Graph and below a Scale-free Network built on the same vertices with the same number of edges.

- The **diameter** is the maximum value of **distance** among all the possible pairs of nodes in the network.
- A **cycle** is a closed **path**, in which the first and last vertices coincide.
- A **tree** is a subgraph of G without **cycles**. If it encompasses all the nodes (but not all the edges) is called a spanning tree. The root is the only vertex with no incoming edges. A **leaf** is a vertex of degree one that is not the **root**.
- A **connected component** in G is a maximal set of firms such that there exists a path between any two of them. We will say that two components are disconnected if there is no path between them. A connected graph is a graph consisting of only one connected component.
- There are several measures of **centrality** that captures in different ways the importance of a node or of an edge. For instance, the **betweenness** centrality of an node capture the number of paths that have to go through node i in order to connect all the pairs of nodes in the network. The feedback centrality is in itself a class of centrality measures that capture the importance of a node in a recursive way, based on a linear combination of the importance of the nodes in its neighborhood. **Eigenvector** centrality belongs to this class and it is the solution, if it is exists unique and positive, of the eigenvalue equation associated with the adjacency matrix, $Ax = \lambda x$.
- The **clustering** coefficient measures the fraction of neighbours of nodes, averaged across the set of nodes, that are also neighbours. In other words, it measures the number of triangles that are realized in the network, relative to the total number of possible triangles that could exist in the network.
- A **community** is an intentionally underspecified notion indicating a group of nodes that are more densely connected among each other than with the nodes that are not in the group.
- **Motifs.** All the possible **graphs** of a given «little» (e.g. 3, 4, 5 order). Their statistics contribute to characterize the topological properties of the network.
- Three classes of network are relevant to this paper, based on the degree distribution:

1. *regular* where directed edges between nodes are assigned randomly under the constraint that all nodes have the same degree k;

2. *random* where the degree distribution follows a Poisson distribution: $\binom{n-1}{k} p^k (1-p)^{n-1-k}$;
3. *scale-free* where the degree distribution follows a power-law distribution: $P(k) \sim k^{-\alpha}$.

• The simplest way to visualize the difference is to look at Figure 7 where two di of graphs are shown for the same set of vertices and the same number of edges.

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